TRANSFORMATION OF NEILL MAPPING FUNCTION FOR GPS TROPOSPHERIC DELAY

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ABSTRACT

The refraction of the Global Positioning System (GPS) signal as it passes through the neutral atmosphere from the satellite to the earth can be referred as GPS tropospheric delay, which causes longer distance traveled by the signal. The tropospheric delay is the product of zenith tropospheric delay and mapping function. Many mapping functions have been established for calculating the tropospheric delay at arbitrary elevation angles. The mapping functions give large value especially for the elevation angles less than 5 degrees. A transformed Neill Mapping Function (NMF) has been proposed to be an alternative mapping function. The result shows a significant reduction of mapping function scale factor. As the coefficient of the zenith tropospheric delay, the value of modified mapping function will directly reduce the total tropospheric delay. The transformed Neill Mapping Function has improved the tropospheric delay up to 17.8 percent for two degrees elevation angles, compared to the Neill Mapping Function.

KEYWORDS*: mapping function, tropospheric, zenith.*

1.0 INTRODUCTION

The issue of atmospheric delay (error) is extensively investigated to minimize the positioning error due to tropospheric and ionospheric delay. Tropospheric delay refers to the refraction of the Global Positioning System (GPS) signal, as it passes through the neutral atmosphere from the satellite to the earth. The effect causes the distance traveled by the signal to be longer than the actual geometric distance between satellite and receiver on the earth. Tropospheric delay can be divided into hydrostatic (dry) delay and wet delay.

At zenith direction, tropospheric delay contributes about 2.5m (Ahn, 2005). Hydrostatic (dry) delay contributes about 90% and wet delay contributes about 10% of the tropospheric delay. This hydrostatic component has a smooth, slowly time-varying characteristic due to its dependence on variations in surface air pressure (weather cells). So this part can be modeled and removed with an accuracy of a few millimeters or better using a surface model (including pressure, temperature and humidity). It does not therefore create much of a problem as far as its effect on GPS signals. Although wet delay is much smaller than the hydrostatic component but the uncertainties in wet tropospheric delay modeling do place a great burden on high precision GPS applications.

The tropospheric delays (TD) at arbitrary elevation angles can be expressed in terms of the zenith delays and mapping functions. This representation allows the use of separate mapping functions for the hydrostatic and wet delay components (Schuler, 2001).

$$TD = ZHD \times m_h(\varepsilon) + ZWD \times m_w(\varepsilon) \tag{1}$$

where :
ZHD is zenith hydrostatic delay (m)
ZWD is zenith wet delay (m)
$m_h(\varepsilon)$ is the hydrostatic mapping function (no unit)
$m_w(\varepsilon)$ is the wet mapping function (no unit)

1.1 Mapping function

Referring to Figure 1, the tropospheric delay is shortest in zenith direction (when the satellite at P) and will become larger with increasing zenith angle (at Q). Projection of zenith path delays into slant direction is performed by application of a *mapping function* or *obliquity factor, m(z)* that is defined as:

$$m(z) = \frac{TD}{ZD} \tag{2}$$

where : TD – total delay (slant neutral delay),
ZD – total zenith delay,
ε - elevation angle (from ground to satellite).
z - zenith angle ($z = 90 - \varepsilon$)

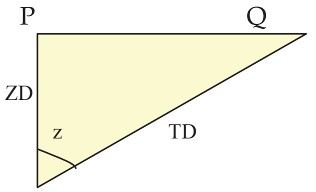

Figure 1 Illustration for obliquity factor (mapping function) between zenith and slant

TD can be separated into two components such as a hydrostatic component (zenith hydrostatic delay, ZHD) and a wet component (zenith wet delay, ZWD) with their mapping function, m(z) as stated in (1) and $ZD = ZHD + ZWD$.

From Figure 1,

$$\frac{ZD}{TD} = \cos z \tag{3}$$

By substitution into (2),

$$m(z) = \frac{1}{\cos z} = \sec z \tag{4}$$

Unfortunately, this secant model is only an approximation assuming a planar surface of the earth and not taking the curvature of the earth into account. Moreover, the temperature and water vapor distribution may cause deviations from this simple formula. Many mapping functions have been established to reduce the delay for arbitrary elevation angles. One of the mapping functions is Neill Mapping Function, which use continued fraction rather than the secant model.

1.2 Neill Mapping Function (NMF)

The mapping functions derived by Arthur Neill (Neill, 1996), are the most widely used, and are known to be the most accurate and easily-implemented functions (Ahn, 2005). In this study, only the Neill mapping function is focused to be modified. Neill Mapping Function states that:

For hydrostatic component,

$$m_h(\varepsilon) = \frac{1+\dfrac{a}{1+\dfrac{b}{1+c}}}{\sin\varepsilon+\dfrac{a}{\sin\varepsilon+\dfrac{b}{\sin\varepsilon+c}}} + \left[\frac{1}{\sin\varepsilon} - \left(\frac{1+\dfrac{a_{ht}}{1+\dfrac{b_{ht}}{1+c_{ht}}}}{\sin\varepsilon+\dfrac{a_{ht}}{\sin\varepsilon+\dfrac{b_{ht}}{\sin\varepsilon+c_{ht}}}}\right)\right] * H \qquad (5)$$

(height correction terms)

for wet component:

$$m_w(\varepsilon) = \frac{1+\dfrac{a_{wet}}{1+\dfrac{b_{wet}}{1+c_{wet}}}}{\sin\varepsilon+\dfrac{a_{wet}}{\sin\varepsilon+\dfrac{b_{wet}}{\sin\varepsilon+c_{wet}}}} \qquad (6)$$

where: ε - elevation angle
m_h - hydrostatic mapping function
m_w - wet mapping function
H - station height above sea level (km).

For the hydrostatic NMF mapping function, the parameter a at tabular latitude ϕ_i at time t from January 0.0 (in UT days) is given as:

$$a(\phi_i, t) = a_{avg}(\phi_i) + a_{amp}(\phi_i)\cos\left(\frac{t - DOY}{365.25}2\pi\right)$$

where DOY (day of year) is the adopted phase, DOY = 28 for Northern hemisphere and DOY = 211 for Southern hemisphere. The linear interpolation between the nearest $a(\phi_i, t)$ is used to obtain the value of $a(\phi_i, t)$. For parameters b and c, a similar procedure was followed.

Height correction coefficients are given as a_{ht} , b_{ht} , c_{ht} and were determined by a least-squares fit to the height correction at nine elevation angles.

The coefficients for the wet NMF mapping function are shown as a_{wet} , b_{wet} , c_{wet} in Appendix 1. No temporal dependence is included in the wet NMF mapping function. Therefore, only an interpolation in latitude for each parameter is required.

2.0 OBJECTIVES

There are two objectives for this study as given below:

2.1 To develop new algorithms of Neill Mapping Function for tropospheric delay (hydrostatic delay model and also wet delay model).

2.2 To investigate the effectiveness of the transformed Neill Mapping Function (NMF) by calculating its mapping function scale factor.

3.0 METHODOLOGY

3.1 Transformation of Neill Mapping Function

The three sine function (denominator) in the first and third terms in equation (5), and also in equation (6) have been rearranged among the sine (s), cosine (c) and also tangent (t) for the hydrostatic and also non-hydrostatic mapping function. The original mapping function using sin, sin and sin (sss) as the denominator of the Neill Mapping Function. For an example, for ssc, it is referring to the first term is s (sine), the second term is s (sine) and the third term c (cosine).

Some rearrangements have been done through the combination of sin, cos and tan for the elevation angles from 2, 3, 4, 5, 10, 45 and 90 degrees. The scale factor of mapping functions for hydrostatic and non hydrostatic components to be calculated using maple software. Only the combination which gives unity at 90 degrees elevation angle and also has smaller value at less than 5 degrees elevation angles will be selected as the alternative mapping function.

Comparison between the current and the selected combinations will be carried out to see the difference. Then, the calculation of reduction percentage will show the improvement of the new mapping function. Both current and new mapping functions will be multiplied by the zenith delay value. The summation of the hydrostatic and non hydrostatic components will give the total tropospheric delay in meter. Finally, the reduction percentage of the modified mapping function compared to the current mapping function will show the improvement of the transformed mapping function.

3.2 Formation of transformed NMF

Calculation of the mapping functions with different combination among sin, cos and tangent have been carried out by using maple software. The purpose of the calculation is to find out which combination will give the smaller scale factor and at the same time to ensure that the value of the mapping function at 90 degrees elevation angle is unity. The calculation has been done for both NMF hydrostatic and also wet components. Finally, the combination of ssc has been selected as the transformed NMF, due to its give smaller value, especially at less than 5 degrees elevation angles.

So, the transformed NMF is based on the equation (5) and (6), where the third sine in the first and the third terms will be replaced by cosine as given in equation (7) and (8) below. The other algorithms and also all coefficients in section 3.1 remain unchanged as given in equation (7) and (8). Transformed Neill Mapping Function is proposed as:

For hydrostatic component,

$$m_h(\varepsilon) = \frac{1+\frac{a}{1+\frac{b}{1+c}}}{\sin\varepsilon+\frac{a}{\sin\varepsilon+\frac{b}{\cos\varepsilon+c}}} + \underbrace{\left[\frac{1}{\sin\varepsilon} - \left(\frac{1+\frac{a_{ht}}{1+\frac{b_{ht}}{1+c_{ht}}}}{\sin\varepsilon+\frac{a_{ht}}{\sin\varepsilon+\frac{b_{ht}}{\cos\varepsilon+c_{ht}}}}\right)\right]}_{\text{(height correction terms)}} *H \qquad (7)$$

for wet component:

$$m_w(\varepsilon) = \frac{1+\frac{a_{wet}}{1+\frac{b_{wet}}{1+c_{wet}}}}{\sin\varepsilon+\frac{a_{wet}}{\sin\varepsilon+\frac{b_{wet}}{\cos\varepsilon+c_{wet}}}} \qquad (8)$$

where: ε - elevation angle
m_h - hydrostatic mapping function
m_w - wet mapping function
H - station height above sea level (km).

4.0 RESULT

4.1 Comparison between hydrostatic NMF and transformed hydrostatics NMF

To compare the effectiveness between the current and the modified combination, the scale factor values for both combination have been shown in the graph below.

4.1.1 Hydrostatic Neill Mapping Function (NMFH)

The ssc combination to be compared with the current mapping function combination (sss) as shown in Table 1 below.

Table 1 Mapping function for hydrostatic component

Elevation angle, ε (degree)	NMFHsss	NMFHssc	Reduction (%)
2	18.4	14.9	18.8
3	14.6	13.5	7.7
4	12.0	11.6	3.4
5	10.1	10.1	0.1
10	5.6	5.6	0.5
45	1.4	1.4	0.0
90	1.0	1.0	0.0

4.1.2 Wet Neill Mapping Function (NMFW)

The ssc combination to be compared with the current mapping function combination (sss) as shown in Table 2

Table 2 Mapping function for wet component

Elevation angle, Ɛ (degree)	NMFWsss	NMFWssc	Reduction (%)
2	21.8	19.6	10.2
3	16.4	15.8	3.6
4	13.0	12.8	1.5
5	10.7	10.5	2.2
10	5.7	5.6	0.5
45	1.4	1.4	0.0
90	1.0	1.0	0.0

4.2 Improvement of Tropospheric Delay (TD)

4.2.1 Current Tropospheric Delay

To calculate the hydrostatic component (A), the current mapping function, NMFH should be multiplied by the ZHD value (2.31m) and the non hydrostatic component (B), the current mapping function, NMFW should be multiplied by the ZWD value (0.25m). The value of ZHD (2.31m) and ZWD (0.25m) are calculated from Saastamoinen model (Saastamoinen, 1972) using Maple software. Calculations for hydrostatic and wet components are shown in Table 3 below:

Table 3 Tropospheric Delay (TD) for the current mapping function

Elev. angle, Ɛ (degree)	A = Current x ZHD (meter)	B = Current x ZWD (meter)	Current TD =A+B (meter)
2	42.58	5.54	48.12
3	33.77	4.17	37.94
4	27.73	3.31	31.04
5	23.39	2.73	26.12
10	12.84	1.44	14.28
45	3.27	0.36	3.63
90	2.31	0.25	2.56

4.2.2 New tropospheric delay

To calculate the hydrostatic component (C), the transformed mapping function, TNMFH should be multiplied by the ZHD value (2.31m) and the non hydrostatic component (D), the transformed mapping function, TNMFW should be multiplied by the ZWD value (0.25m) as shown in Table 4 below:

Table 4 New Tropospheric Delay (TD) for the transformed NMF (TNMF)

Elev. angle, Ɛ (degree)	C = TNMFH x ZHD (meter)	D = TNMFW x ZWD (meter)	New TD = C + D (meter)
2	34.57	4.98	39.55
3	31.16	4.02	35.18
4	26.78	3.26	30.04
5	23.38	2.67	26.05
10	12.88	1.43	14.31
45	3.27	0.36	3.63
90	2.31	0.25	2.56

4.3 Improvement percentage for the new tropospheric delay

Comparison for the current and the transformed mapping function will also show the comparison of the current and the new tropospheric delay. The improvement of the new tropospheric delay can be shown in Table 5.

Table 5 Percentage improvement for the tropospheric delay

Elevation angle, Ɛ (degree)	Current TD (meter)	New TD (meter)	Improvement (%)
2	48.12	39.55	17.8
3	37.93	35.17	7.3
4	31.04	30.04	3.2
5	26.12	26.04	0.3
10	14.31	14.31	0.0
45	3.63	3.63	0.0
90	2.57	2.57	0.0

5.0 CONCLUSION

The study shows that the combination of sine, sine and cosine (ssc) is a suitable choice to be an alternative for the mapping function . For the elevation angles less than 5 degrees, the reduction percentage of the scale factor is very obvious. After the transfromation, the reduction percentage for hydrostatic and wet components can give the improvement percentage up to 18.8 and 10.2 percent respectively. On the other hand, the reduction shows the significant improvement of the mapping function, either for hydrostatic or non hydrostatic mapping functions. As the coefficient to the zenith tropospheric delay, the improvement of mapping function can directly reduce the total tropospheric delay of the GPS signal. The improvement of the tropospheric delay is 17.8 percent for two elevation angles.

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7.0 REFERENCES

Ahn, Y.W (2005). Analysis of NGS CORS Network for GPS RTK Performance Using External NOAA Tropospheric Corrections Integrated with a Multiple Reference Station Approach, *M.Sc. Theses*, University of Calgary, Canada.

Niell, A. E. (1996). Global mapping functions or the atmosphere delay at radio wavelengths, *Journal of Geophysical Resarch, 101,* pp 3227- 3246.

Saastamoinen, J. (1972). Atmospheric correction for troposphere and stratosphere in radio ranging of satellites, Geophysical monograph, 15, *American Geophysical Union*, Washington, D. C., USA, pp. 247-252.

Schuler, T. (2001). On Ground-Based GPS Tropospheric Delay Estimation, *PhD Thesis,* University of Munchen.

Appendix 1 NMF coefficients for hydrostatic and wet

coefficient	$\phi_i = 15$	$\phi_i = 30$	$\phi_i = 45$	$\phi_i = 75$	$\phi_i = 90$
a_{avg}	$1.2769934 \cdot 10^{-3}$	$1.2683230 \cdot 10^{-3}$	$1.2465397 \cdot 10^{-3}$	$1.2196049 \cdot 10^{-3}$	$1.2045996 \cdot 10^{-3}$
b_{avg}	$2.9153695 \cdot 10^{-3}$	$2.9152299 \cdot 10^{-3}$	$2.9288445 \cdot 10^{-3}$	$2.9022565 \cdot 10^{-3}$	$2.9024912 \cdot 10^{-3}$
c_{avg}	$62.610505 \cdot 10^{-3}$	$62.837393 \cdot 10^{-3}$	$63.721774 \cdot 10^{-3}$	$63.824265 \cdot 10^{-3}$	$62.258455 \cdot 10^{-3}$
a_{amp}	0.0	$1.2709626 \cdot 10^{-5}$	$2.6523662 \cdot 10^{-5}$	$3.4000452 \cdot 10^{-5}$	$4.1202191 \cdot 10^{-5}$
b_{amp}	0.0	$2.1414979 \cdot 10^{-5}$	$3.0160779 \cdot 10^{-5}$	$7.2562722 \cdot 10^{-5}$	$11.723375 \cdot 10^{-5}$
c_{amp}	0.0	$9.0128400 \cdot 10^{-5}$	$4.3497037 \cdot 10^{-5}$	$84.795348 \cdot 10^{-5}$	$170.37206 \cdot 10^{-5}$
a_{ht}	$2.53.10^{-5}$				
b_{ht}	$5.49.10^{-3}$				
c_{ht}	$1.14.10^{-3}$				
a_{wet}	$5.8021897.10^{-4}$	$5.6794847 \cdot 10^{-4}$	$5.8118019 \cdot 10^{-4}$	$5.9727542 \cdot 10^{-4}$	$6.1641693 \cdot 10^{-4}$
b_{wet}	$1.4275268.10^{-3}$	$1.5138625 \cdot 10^{-3}$	$1.4572752 \cdot 10^{-3}$	$1.5007428 \cdot 10^{-3}$	$1.7599082 \cdot 10^{-3}$
c_{wet}	$4.3472961 \cdot 10^{-2}$	$4.6729510 \cdot 10^{-2}$	$4.3908931 \cdot 10^{-2}$	$4.4626982 \cdot 10^{-2}$	$5.4736038 \cdot 10^{-2}$