# SIMPLER NONHYDROSTATICS UNBAB MAPPING FUNCTION FOR TROPOSPHERIC DELAY 

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#### Abstract

There are seven mathematical operations to calculate the mapping function values for nonhydrostatics UNBab mapping function. This mapping function is chosen to be simplified due to its ability to calculate mapping function value down to two (2) degree of elevation angle. To simplify the model, regression method is used in order to get the similar result. The sum of errors calculation shows that the deviation of the simplified model from the original models is not significant.


KEYWORDS: Tropospheric delay; Zenith; Mapping function.

### 1.0 INTRODUCTION

In 1972, the founder of mapping function, Marini states that the elevation angle $E$ dependence of any horizontally stratified atmosphere can be approximated by expanding in a continued fraction in term of 1 / sin $E($ Marini, 1972). The total tropospheric delay values can be obtained by multiplying the mapping function values with zenith hydrostatic delay and also zenith non hydrostatic delay as given in equation (1) below (Schuler, 2001):

$$
\begin{equation*}
T D=Z H D . m_{h}(E)+Z W D . m_{w}(E) \tag{1}
\end{equation*}
$$

where:
ZHD = zenith hydrostatic delay (m)
ZWD = zenith wet delay ( m )
$m_{h}(E)=$ hydrostatic mapping function
$m_{h}(E)=$ non-hydrostatic mapping function
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### 2.0 SIMPLIFICATION PROCESS OFNONHYDROSTATICS UNBAB MAPPING FUNCTION

In Guo (2003), a researcher from the University of New Brunswick, has established the mapping function model. The model has 7 operations in a form of continued fraction. The nonhydrostatic component of is written as:

$$
\begin{equation*}
\operatorname{UNBab}_{n h}(E)=\frac{1+\frac{a_{n h}}{1+b_{n h}}}{\sin E+\frac{a_{n h}}{\sin E+b_{n h}}} \tag{2}
\end{equation*}
$$

where, $E$ : elevation angle (degree)
The parameters $a_{n h}$ and $b_{n h}$ for the hydrostatic functions are:

$$
\begin{aligned}
a_{n h} & =(0.73537-0.041172 \mathrm{H}-0.00202 \cos \phi) / 1000 . \\
b_{n h} & =(32.5627-0.670636 \mathrm{H}-0.15502 \cos \phi) / 1000 .
\end{aligned}
$$

where $\mathrm{H}=0.1 \mathrm{~km}$ and $\phi$ is 45 degrees.
The mapping function for the original $U_{N B a b_{n k}(E), ~ n a m e d ~ a s ~} S$ shown in Figure 1.


Figure 1. Graph of mapping functions for $S$
The mapping function give a shape of hyperbola and equation (2) can be simplified as:

$$
\begin{equation*}
S_{1}(E)=A E^{B} \tag{3}
\end{equation*}
$$

where

$$
\begin{align*}
& S_{1}(E)=\text { simplified } U N B a b_{n h}(E) \\
& A, B=\text { constant } \\
& E=\text { elevation angle (degree) } \\
& \qquad \log _{10} S_{1}=B \log _{10} E+\log _{10} A \tag{4}
\end{align*}
$$

Therefore, equation (3) has changed the shape from hyperbolic graph into linear graph as shown in equation (4) The gradient of $B$ and the graph will intersect the axis at as shown in Figure 2 below.


Figure 2. Graphs of $\mathrm{S}, \mathrm{S}_{1}$ and $\mathrm{S}_{2}$ for $U N B a b_{n h}(E)$ in logarithm scale
By linear regression method, equation (4) becomes:

$$
\begin{equation*}
\log _{10} S_{1}=-0.8924 \log _{10} E+1.6605 \tag{5}
\end{equation*}
$$

where, $\mathrm{B}=-0.8924$ and $\log _{10} A=1.6605$ which gives $A=45.761$.

Therefore, equation (5) becomes:

$$
\begin{equation*}
S_{1}(E)=45.761 E^{-0.8924} \tag{6}
\end{equation*}
$$

By regression method, polynomial equation, $\log S_{2}$ can be generated from the original model $(\log S)$ in a form of quadratic equation as given below:

$$
\begin{equation*}
\log _{10} S_{2}=0.1575\left(\log _{10} E\right)^{2}-1.2762\left(\log _{10} E\right)+1.852 \tag{7}
\end{equation*}
$$

The graphs of $S, S_{1}$ and $S_{2}$ can be shown in Figure 3.


Figure 3. Graphs of $S, S_{1}$ and $S_{2}$ for $U N B a b_{n h}(E)$ mapping function

### 3.0 CALCULATION OF SUM OF ERROR (SOE) FOR UNBAB NH (E)

Sum of error (SOE) method can be used to show how the simplified models deviate from the original model. Smaller deviation is better, which shows that the simplified model is closer to the original model.

Table 1. Sum of error between $S$ and simplified models $S_{1} \& S_{2}$

| $\mathbf{E}$ | $\mathbf{S}$ | $\mathbf{S 1}$ | $\mathbf{S 2}$ | $\mathbf{( S - S 1 ) ^ { \wedge } \mathbf { 2 }}$ | $(\mathbf{S - S 2})^{\wedge} \mathbf{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 28.636 | 26.607 | 27.849 | 4.118 | 0.621 |
| 5 | 11.471 | 11.482 | 10.602 | 0.000 | 0.756 |
| 10 | 5.758 | 6.080 | 5.413 | 0.104 | 0.120 |
| 20 | 2.924 | 3.220 | 2.906 | 0.088 | 0.000 |
| 30 | 2.000 | 2.220 | 2.067 | 0.048 | 0.004 |
| 40 | 1.556 | 1.705 | 1.640 | 0.022 | 0.007 |
| 50 | 1.305 | 1.389 | 1.379 | 0.007 | 0.005 |
| 60 | 1.155 | 1.175 | 1.201 | 0.000 | 0.002 |
| 70 | 1.064 | 1.020 | 1.072 | 0.002 | 0.000 |
| 80 | 1.015 | 0.903 | 0.973 | 0.013 | 0.002 |
| 90 | 1.000 | 0.810 | 0.895 | 0.036 | 0.011 |
|  |  |  | SOE | $\mathbf{4 . 4 3 8}$ | $\mathbf{1 . 5 2 9}$ |
|  |  |  |  |  |  |

Table 1 shows that the sum of error is very small, which is closer to the original model.

Therefore, the difference of mapping function between the original model and the simplified model is not significance.

### 4.0 COMPUTATION TIME FOR $\operatorname{UNBABNH}(E)$

The computation time for calculating ( 100,000 cycles) the original model and also the simplified model for can be shown using CodeGear C++ Builder 2007 software (Hamzah, 2008).

Table 2. Comparison for the computation time

| Model | Computation time for <br> original model, $\mathrm{S}(\mathrm{ms})$ | Computation time for <br> simplified model, $\mathrm{S}_{1}(\mathrm{~ms})$ | Reduction <br> (times) | of computation | time |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $U N B a b_{n h}$ | 244.3 | 63.1 |  | 3.9 |  |

Table 2 shows that the computation time between the original model and modified $U N B a b_{n k}(E)$ model shows that the modified model is 3.9 times faster than the original model.

Table 3. Reduction percentage of the number of model operations

| Model | Number of operations <br> (Original model, S) | Number of operations <br> $\left(\right.$ Simplified model, $\left.S_{1}\right)$ | Operation <br> reduction | Percentage of <br> reduction |
| :---: | :---: | :---: | :---: | :---: |
| $U N B a b_{n h}$ | 7 | 2 | 5 | 71.4 |

From Table 3, the simplified $\operatorname{UNBab}_{n h}(E)$ model can reduce the number of operations up to 71.4 percent compared to the original model.

### 5.0 CONCLUSION

Based on sum of error (SOE) result, the simplified models, $S_{1}$ and $S_{2}$ are only 4.438 and 1.529 respectively which are very small and not significant compare to the original model, S . The computation time for the simplified model, $\mathrm{S}_{1}$ is reduced, which is 3.9 times faster than the original model. Lastly, the number of model operations of the simplified model is reduced 71.4 percent from the original model, S. Based on the results above, the simplified $\operatorname{UNBab}_{n k}(E)_{1}$ mapping function model can be used to replace the original model due to its simpler model, smaller sum of error values and also the shorter computation time compared to the original model.

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