

APPLICATION OF VIM, HPM AND CM TO THE SYSTEM OF STRONGLY NONLINEAR FIN PROBLEM

M. R. Shirkhani¹, H.A. Hoshyar¹, D.D. Ganji¹

¹Department of Mechanical Engineering, Sari Branch,
Islamic Azad University, Sari, Iran.

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ABSTRACT

The nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient is analytically studied. Collocation method (CM), Variation iteration method (VIM) and Homotopy Perturbation Method (HPM) are used to solve the present problem. Also, fourth order Runge–Kutta numerical method is applied as a numerical method for validation. Analytical results are presented through the graphs and the tables in various values of parameters. The results reveal that the CM is very effective, simple and more accurate than other techniques. Furthermore, we analyze the effects of some physical applicable parameters in this problem such as thermal conductivity parameter (β), thermo-geometric fin parameter (M) and heat transfer mode (m).

KEYWORDS: *Collocation method (CM); variational iteration method (VIM); homotopy perturbation method (HPM); heat transfer; fin*

1.0 INTRODUCTION

Fins or extended surfaces are frequently used to enhance the heat transfer between a solid surface and its surrounding medium. Extend surfaces are extensively used in various industrial applications; for example, liquid-gas heat exchangers, air-cooled internal combustion engines, the electrical apparatus, nucleate boiling, etc. Since Most of problems and scientific phenomenon such as heat transfer problems for the fins (Kiwani, 2007; Gorla Reddy & Bakier, 2011; Domairry & Fazeli, 2011; Ganji, 2011; Khani et al, 2009), are inherently of nonlinearity. Therefore, the differential equation for a convective fin does not admit an exact solution and these nonlinear equations should be solved using other methods, Such as numerical analysis or analytical method. In the analytical perturbation method, we should exert a small parameter in the

* Corresponding Email: hoshyarali@gmail.com

equation. Therefore, finding this parameter and exerting it into the equation is difficulties of this method. In recent years, scientists have presented some new methods for solving nonlinear differential equations; for instance, δ -expansion method (Ganji & Hashemi, 2011a), Adomian's decomposition method (Ganji & Hashemi, 2011b), Homotopy perturbation method (HPM) (He, 1999; He 2005a; He, 2005b; Esmailpour & Ganji, 2007; He, 2010; Ganji & Rajabi, 2006; Ganji, Ganji & Ganji, 2011) and Variational iteration method (VIM) (He, 2007; He & Wu, 2006; Ganji, Rostamiyan, Petroudi & Nejad, 2014; Ganji, Tari & Jooybari, 2014; He, 1999). One of the other semi-exact methods is the weighted residual methods (WRMs). Collocation method (CM), Galerkin method (GM), and least square method (LSM) are examples of WRMs. These methods are the most effective and convenient ones for both linear and nonlinear equations. Stern and Rasmussen used collocation method for solving a third order linear differential equation (Stern & Rasmussen, 1996). Hu and Li and Herrera et al. applied collocation method for Poisson's equation and advection-diffusion equation respectively (Hu & Li, 2006; Herrera, Diazviera, Yates, 2004). Arnau et al., presented a new alternative method based in polynomial collocation and used it to model the flow in intake and exhaust of internal combustion engine. Hendi and Albugami solved Fredholm-Volterra integral equation using collocation method (Hendi & Albugami, 2010). Recently Hatami and Ganji applied CM on non-Newtonian nanofluid passing through the porous media between two coaxial cylinders (Hatam & Ganji, 2013). In this article, the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient is solved through the three methods: Collocation method, Homotopy perturbation method and the Variation iteration method. Also Runge-Kutta method is used to evaluate the excellence and accuracy of the proposed methods. The computations show that CM allows us to obtain approximations with an error relative to the numerical solution smaller than the errors obtained using other methods.

2.0 METHODOLOGY

2.1 Governing equations

Consider a straight fin with an arbitrary constant cross-sectional area; perimeter and length. The fin is attached to a base surface of temperature, extends into a fluid of temperature, and its tip is insulated. The one-dimensional energy balance equation is given as in Equation (1) below: -

$$A_c \frac{d}{dX} \left(k \frac{dT}{dX} \right) + ph(T - T_a) = 0, \quad (1)$$

$$0 < X < L$$

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to the expression in Equation (2):-

$$k = k(T) = k_a [1 + \lambda(T - T_a)] \quad (2)$$

where k_a is the thermal conductivity at the ambient fluid temperature of the fin and λ is the parameter describing the thermal conductivity variation. Also It is considered that h vary with temperature function by Equation (3):-

$$h = h(T) = h_b \left(\frac{T - T_a}{T_b - T_a} \right)^m \quad (3)$$

where h_b is the heat transfer coefficient at the base temperature. The exponent m depends on the heat transfer mode. Typical values of n are $-1/4$ for laminar film boiling or condensation, 0 constant for heat transfer coefficient, $1/4$ for laminar natural convection, $1/3$ for turbulent natural convection, 2 for nucleate boiling, and 3 for radiation.

In order to simplify the energy equation, the dimensionless parameters are defined as in Equation (4) as follow: -

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad x = \frac{X}{L}, \quad k = \frac{k}{k_a}, \quad \beta = \lambda(T_b - T_a), \quad M^2 = \frac{h_b pl^2}{k_a A_c} \quad (4)$$

Hence, the energy in Equation (1) will take the form (Khani, Raji & Nejad, 2009) as in Equation (5) below: -

$$\frac{d}{dX} \left[k(\theta) \frac{d\theta}{dX} \right] = M^2 \theta^{m+1}, \quad 0 < x < 1 \quad (5)$$

The boundary conditions are as given in Equation (6) below: -

$$\begin{aligned} \theta'(0) &= 0 \quad \text{at the tip,} \\ \theta(1) &= 1 \quad \text{at the base} \end{aligned} \tag{6}$$

The governing Equation (5) can be rewritten as in Equation (7) below:-

$$(1 + \beta\theta)\theta'' - M^2 \theta^{m+1} + \beta(\theta')^2 = 0 \tag{7}$$

2.2 Basic idea of Collocation method

Collocation method is one of the approximation techniques for solving differential equations called the Weighted Residual Methods (WRMs). For the conception of the main idea of this method, suppose a differential operator D is acted on a function u to produce a function p (Hatami and Ganji, 2013) as in Equation (8) below: -

$$D(u(x)) = p(x) \tag{8}$$

We wish to approximate u by a function \tilde{u} , which is a linear combination of basic functions chosen from a linearly independent set, as in Equation (9):-

$$u = \tilde{u} = \sum_{i=1}^n c_i \varphi_i \tag{9}$$

Now, when substituted into the differential operator, D , the result of the operations is not $p(x)$. Hence an error or residual will exist as shown in Equation (10) below: -

$$E(x) = R(x) = D(\tilde{u}(x) - p(x)) \neq 0 \tag{10}$$

The notion in the Collocation is to force the residual to zero in some average sense over the domain, as given in Equation (11) below: -

$$\int_x R(x) W_i(x) dx = 0, \quad i = 1, 2, \dots, n \tag{11}$$

where the number of weight functions W_i is exactly equal the number of unknown constants c_i in \tilde{u} . The result is a set of n algebraic equations for the unknown constants c_i . For Collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $w_i(x) = \delta(x-x_i)$. The Dirac δ function has the property as shown in Equation in (12) below:-

$$\delta(x-x_i) = \begin{cases} 1 & \text{if } x = x_i \\ 0 & \text{other wise} \end{cases} \quad (12)$$

The residual function in Equation (10) must be forced to be zero at specific points.

2.3 Application of collocation method

We wish to obtain an approximate solution for this problem in the interval $0 < x < 1$.

To construct a trial solution, we choose the basic function to polynomial in. The trial solution contains four undetermined coefficients and satisfies the conditions for all values as given in Equation (13) below: -

$$\theta(x) = 1 + c_1(1-x^2) + c_2(1-x^3) + c_3(1-x^4) + c_4(1-x^5) \quad (13)$$

Whereas the trial solution satisfies the boundary condition of Equation (6). The accuracy of the solution can be improved by increasing the number of its terms. When θ is introduced into differential equation it yields residual $R(x)$ as in Equation (14) as follows:-

$$\begin{aligned} R(x) = & -2c_1 + 6\beta c_1^2 x^2 - 20\beta c_4^2 x^3 + 28\beta c_3^2 x^6 - 2\beta c_3 c_1 - 2\beta c_4 c_1 - 6\beta c_2 x \\ & - 12c_3 x^2 - M^2(1 + c_1 - c_1 x^2 + c_2 - c_2 x^3 + c_3 - c_3 x^4 + c_4 - c_4 x^5)^m c_2 + \dots \\ & - M^2(1 + c_1 - c_1 x^2 + c_2 - c_2 x^3 + c_3 - c_3 x^4 + c_4 - c_4 x^5)^m c_4 x^5 \end{aligned} \quad (14)$$

Now the problem of finding approximate solution of the problem in the interval $0 < x < 1$ becomes one adjusting the values of c_1, c_2, c_3 and c_4 . So that residual stays close to zero throughout the interval $0 < x < 1$. The basic assumption is that the residual does not deviate much from zero between collocation locations. For reaching to this aim, four specific points should be chosen. These points are as given in Equations (15) to (19) as follow:-

$$R\left(\frac{1}{5}\right) = 0, \quad R\left(\frac{2}{5}\right) = 0, \quad R\left(\frac{3}{5}\right) = 0, \quad R\left(\frac{4}{5}\right) = 0, \quad (15)$$

$$\begin{aligned} R\left(\frac{1}{5}\right) = & -2c_1 - \frac{6}{5}c_2 - \frac{12}{25}c_3 - \frac{4}{25}c_4 - \frac{304}{125}\beta c_3 c_1 - \frac{6708}{3125}\beta c_4 c_1 - \frac{76}{25}\beta c_2 c_1 \\ & - \frac{21194}{15625}\beta c_2 c_4 - \frac{124}{125}M^2\left(1 + \frac{24}{25}c_1 + \frac{124}{125}c_2 + \frac{624}{625}c_3 + \frac{3124}{3125}c_4\right)^m c_2 + \dots \\ & - \frac{12491}{78125}\beta c_4^2 - M^2\left(1 + \frac{24}{25}c_1 + \frac{124}{125}c_2 + \frac{624}{625}c_3 + \frac{3124}{3125}c_4\right)^m = 0 \end{aligned} \quad (16)$$

$$\begin{aligned}
 R\left(\frac{2}{5}\right) = & -2c_1 - \frac{12}{5}c_2 - \frac{48}{25}c_3 - \frac{32}{25}c_4 - \frac{394}{125}\beta c_3c_1 - \frac{8906}{3125}\beta c_4c_1 - \frac{78}{25}\beta c_2c_1 \\
 & - \frac{53916}{15625}\beta c_2c_4 - \frac{117}{125}M^2\left(1 + \frac{21}{25}c_1 + \frac{117}{125}c_2 + \frac{609}{625}c_3 + \frac{3093}{3125}c_4\right)^m c_2 + \dots \\
 & - \frac{97696}{78125}\beta c_4^2 - M^2\left(1 + \frac{21}{25}c_1 + \frac{117}{125}c_2 + \frac{609}{625}c_3 + \frac{3093}{3125}c_4\right)^m = 0
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 R\left(\frac{3}{5}\right) = & -2c_1 - \frac{18}{5}c_2 - \frac{108}{25}c_3 - \frac{108}{25}c_4 - \frac{304}{125}\beta c_3c_1 - \frac{9544}{3125}\beta c_4c_1 - \frac{32}{25}\beta c_2c_1 \\
 & - \frac{82926}{15625}\beta c_2c_4 - \frac{98}{125}M^2\left(1 + \frac{16}{25}c_1 + \frac{98}{125}c_2 + \frac{544}{625}c_3 + \frac{2882}{3125}c_4\right)^m c_2 + \dots \\
 & - \frac{278451}{78125}\beta c_4^2 - M^2\left(1 + \frac{16}{25}c_1 + \frac{98}{125}c_2 + \frac{544}{625}c_3 + \frac{2882}{3125}c_4\right)^m = 0
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 R\left(\frac{4}{5}\right) = & -2c_1 - \frac{24}{5}c_2 - \frac{192}{25}c_3 - \frac{256}{25}c_4 - \frac{326}{125}\beta c_3c_1 - \frac{4758}{3125}\beta c_4c_1 - \frac{86}{25}\beta c_2c_1 \\
 & - \frac{5624}{15625}\beta c_2c_4 - \frac{61}{125}M^2\left(1 + \frac{9}{25}c_1 + \frac{61}{125}c_2 + \frac{369}{625}c_3 + \frac{2101}{3125}c_4\right)^m c_2 + \dots \\
 & - \frac{210176}{78125}\beta c_4^2 - M^2\left(1 + \frac{9}{25}c_1 + \frac{61}{125}c_2 + \frac{369}{625}c_3 + \frac{2101}{3125}c_4\right)^m = 0
 \end{aligned} \tag{19}$$

Thus we can obtain coefficient for different value of β , M and m . For example, Using Collocation method with $\beta = 0.1$, $M = 1$, $m = 2$, $\theta(x)$ is as expressed in Equation (20) below:-

$$\theta(x) = 0.7634481393 + 0.2024689517x^2 + 0.0141005165x^3 + 0.0028024632x^4 + 0.0171799293x^5 \tag{20}$$

2.4 Variational iteration method

To illustrate the basic idea of variational iteration method, we consider the following general nonlinear system, as given in Equation (21): -

$$Lu + Nu = g(t) \tag{21}$$

where L is a linear operator, N nonlinear operator, $g(t)$ a homogeneous term. According to the variational iteration method, we can construct the following iteration using the formulation in Equation (22) below: -

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda [Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)] d\tau \tag{22}$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory. The subscript n indicates the n th approximation and \tilde{u}_n is considered as a restricted variation, i.e., $\delta\tilde{u}_n = 0$.

2.5 Application of Variational iteration method

First we construct a correction functional which reads an expression as in Equation (23) as follows:-

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \lambda \left\{ [1 + \beta \theta_n(\tau)] \theta_n''(\tau) - M^2 \theta_n^{m+1}(\tau) + \beta [\theta_n'(\tau)]^2 \right\} d\tau \tag{23}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition. For example, when $M = 1$, $\beta = -0.4$ and $m = 0$, $\theta_0(x)$ is given as in Equation (24) below:-

$$\theta_0(x) = \frac{e^{-x}}{e^{-1} + e} + \frac{e^{-x}}{e^{-1} + e} \tag{24}$$

Also, making the above correction functional stationary, we can obtain following stationary conditions, as given in Equation (25) below: -

$$\lambda'(t) - \lambda(t) = 0, \quad 1 - \lambda'(t) \Big|_{t=x} = 0, \quad \lambda(t) \Big|_{t=x} = 0 \tag{25}$$

The Lagrangian multiplier can therefore be identified as in Equation (26) below: -

$$\lambda = -\frac{1}{2} (e^{(x-t)} - e^{(t-x)}) \tag{26}$$

Substituting $\theta_0(x)$ and λ into Equation (23) and after some simplifications, we have the following expressions as in Equations (27) and (28) :

$$\begin{aligned} \theta_1(x) = c_0 [& 0.1192029221 e^{-x+1} + 0.1192029221 e^{x+1} - 0.003337477915 e^x \\ & - 0.003337477915 e^{x+2} - 0.003337477915 e^{-x} + 0.002503108436 e^{2x} - 0.003337477915 e^{-x+2} \\ & - 0.003337477915 e^{2x} + 0.003337477915 e^{2x+2} + 0.003337477915 e^{-2x} + 0.003337477915 e^{-2x+2}] \end{aligned} \tag{27}$$

where $c_0 = \frac{1}{A}$, that $A = 1.124262852$

$$\begin{aligned} \theta_2(x) = c_1 [& -0.0000035250 \sinh(x)x + 0.106027626 e^{-x+1} + 0.106027626 e^{x+1} \\ & - 0.002968592 e^x - 0.002968592 e^{x+2} - 0.002968592 e^{-x} - 0.002968592 e^{-x+2} \\ & + \dots + 0.000141638 \cosh(3x+3) + 0.000141638 \cosh(3x-3) - 0.00000396564 \cosh(3x+4)] \end{aligned} \tag{28}$$

where $c_1 = \frac{1}{B}$, that $B = 1.031737135$

In a similar manner, we will obtain other solutions for different cases M , β and m . The results are presented graphically.

2.6 Analysis of He’s Homotopy perturbation method

To explain the basic ideas of this method, the following expressions in Equations (29) to (30) can be considered:-

$$A(u) - f(r) = 0, r \in \Omega \tag{29}$$

with the boundary condition of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma \tag{30}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω . A can be divided into two parts, which are L and N , where L is linear and N is nonlinear Equation (1) can therefore be rewritten as given in Equation (31) below:-

$$L(u) + N(u) - f(r) = 0, r \in \Omega \tag{31}$$

Homotopy perturbation structure is given in Equations (32) and (33) as follows:-

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{32}$$

where

$$v(r, p): \Omega \times [0, 1] \rightarrow R \tag{33}$$

In Equation (4), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. It can be assumed the solution of Equation (4) can be written as a power series in p , as in Equation (34) below:-

$$v = v_0 + p^1 v_1 + p^2 v_2 + \dots = \sum_{i=0}^n v_i p^i \tag{34}$$

and the best approximation for the solution is given in Equation (35) as below:-

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \tag{35}$$

2.7 Application of Homotopy perturbation method

In this section, we will apply the HPM to nonlinear ordinary differential system (1). according to Equation (7), Using HPM, when $M = 1$, $\beta = -0.4$ and $m = 0$ leads to an expression in Equation (36) below:-

$$H(\theta, p) = (1 - P) [\theta''(x) - \theta(x)] + p [(1 - 0.1\theta)\theta'' - \theta - 0.1(\theta')^2] = 0 \tag{36}$$

We consider $\theta(x)$ as an expression in Equation (37) below:-

$$\theta(x) = \theta_0(x) + p\theta_1(x) + p^2\theta_2(x) \tag{37}$$

By substituting $\theta(x)$ from Equation (37) into Equation (36) and after some simplifications and rearrangements based on powers of p -terms, we have the following expressions as shown in Equations (38) to (40):-

$$p^0: \begin{aligned} \theta_0''(x) - \theta_0(x) &= 0 \\ \theta_0(1) = 1, \theta_0'(0) &= 0 \end{aligned} \tag{38}$$

$$p^1: \begin{aligned} \theta_1''(x) - 0.4[\theta_0'(x)]^2 - \theta_1(x) - 0.4\theta_0(x)[\theta_0'(x)] &= 0 \\ \theta_1(1) = 0, \theta_1'(0) &= 0 \end{aligned} \tag{39}$$

$$p^2: \begin{aligned} \theta_2''(x) - \theta_2(x) - 0.4\theta_1(x)\theta_0''(x) - 0.4\theta_0(x)\theta_1''(x) - 0.8\theta_1'(x)\theta_0'(x) &= 0 \\ \theta_2(1) = 0, \theta_2'(0) &= 0 \end{aligned} \tag{40}$$

Solving Equations (38) to (40) with boundary conditions, we have the following expressions given in Equations (41) to (44):-

$$\theta_0(x) = \frac{e^{-x}}{e^{-1} + e} + \frac{e^x}{e^{-1} + e} \tag{41}$$

$$\theta_1(x) = -\frac{4}{15} \left[\frac{e^x(e^4 + 1)}{2e^{-1}e^2 + e^{-1} + e^{-1}e^4 + e e^4 + 2e e^2 + e} + \frac{e^{-x}(e^4 + 1)}{2e^{-1}e^2 + e^{-1} + e^{-1}e^4 + e e^4 + 2e e^2 + e} - \frac{e^{-2x+2}(e^4x + 1)}{e^4 + 30e^2 + 15} \right] \tag{40}$$

$$\theta_2(x) = \frac{1}{225} \left[\frac{e^{-x}(76e^5 + 5e + 5e^9 - 33e^7 - 9e^3)e^{-1}}{6e^{-1}e^2 + 4e^{-1}e^6 + e^{-1} + 4e^{-1}e^2 + e^{-1}e^8 + 4e e^2 + e + e e^8 + 6e e^4 + 4e e^6} + \dots + \frac{e^{-3x+2}}{e^8 + 4e^2 + 1 + 6e^4 + 4e^6} \left[(-32e^x - 32e^{5x+4} + 27e^{6x+1} 12x e^{4x+1} + \dots - 12x e^{2x+3}) \right] \right] \tag{44}$$

The solution of this equation, when $p \rightarrow 1$, will be as follows:

$$\theta(x) = \theta_0(x) + \theta_1(x) + \theta_2(x) \tag{45}$$

In a similar manner, we will obtain other solutions for different cases M , β and m . The results are presented in next section.

3.0 RESULTS AND DISCUSSION

In this manuscript the collocation method such as analytical technique is employed to find an analytical solution of the nonlinear fin problem. The results are compared with other analytical methods such as VIM and HPM. For validation all these results are compared with the numerical solution. Figures 1-3 and Table 1 show the temperature distribution with the axial distance along the fin for the three methods. It is interesting to note that collocation method is very close to the numerical results and the results of HPM and VIM are significantly in error.

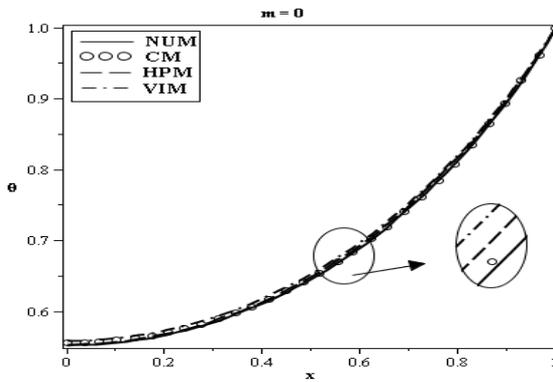


Figure 1. Comparison between the CM, VIM, HPM and numerical solution for $m = 0, M = 1, \beta = -0.4$

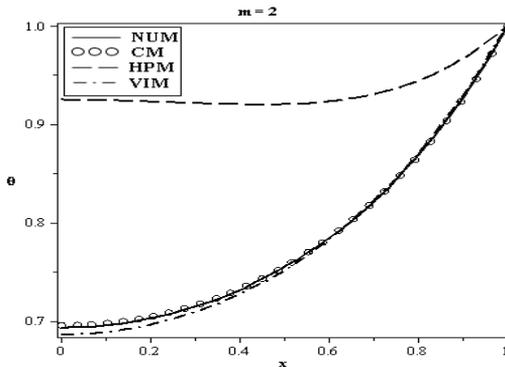


Figure 2. Comparison between the CM, VIM, HPM and numerical solution for $m = 2, M = 1, \beta = -0.4$

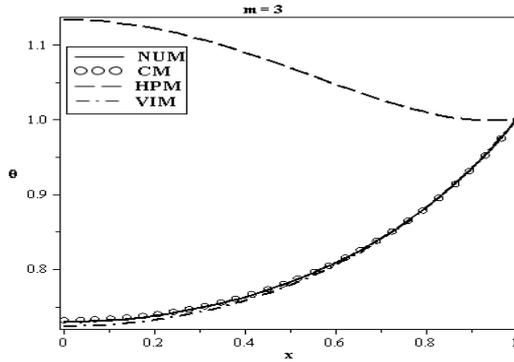


Figure 3. Comparison between the CM, VIM, HPM and numerical solution for $m = 3, M = 1, \beta = -0.4$

Table 1 The results of CM, VIM, HPM, and numerical methods for $m = 2, M = 1, \beta = -0.4$

x	CM	VIM	HPM	NUM	ERROR VIM	ERROR HPM	ERROR CM
0	0.69636640	0.68655496	0.9250000	0.69340454	0.00684958	0.23159545	0.00296185
0.1	0.6984550	0.68900216	0.92451250	0.69571666	0.00671450	0.22879583	0.00273842
0.2	0.7050589	0.69641743	0.92320000	0.70271768	0.00630025	0.22048231	0.00234130
0.3	0.7165880	0.70902464	0.92151250	0.71460737	0.00558272	0.20690512	0.00198068
0.4	0.7334326	0.72720630	0.92020000	0.73173911	0.00453281	0.18846088	0.00169357
0.5	0.7561004	0.75151812	0.92031250	0.75465474	0.00313661	0.16565775	0.00144573
0.6	0.7853530	0.78271046	0.92320000	0.78414305	0.00143259	0.13905694	0.00121000
0.7	0.8223430	0.82175744	0.93051250	0.82133639	0.00042104	0.10917610	0.00100665
0.8	0.8687507	0.86989507	0.94420000	0.86787250	0.00202256	0.07632749	0.00087821
0.9	0.9269209	0.92866991	0.96651250	0.92617822	0.00249169	0.04033427	0.00074272

Figures 4, 5 and 6 show comparison between the numerical solutions and CM solutions in predicting θ for different values of thermal conductivity (β), thermo-geometric fin parameter (M) and heat transfer mode (m) respectively. We can see a very good agreement between the collocation and the numerical results. In addition, It can be seen in these figures that by increasing the β and m , velocity profiles increases, but the velocity profiles decreases with the increase in M .

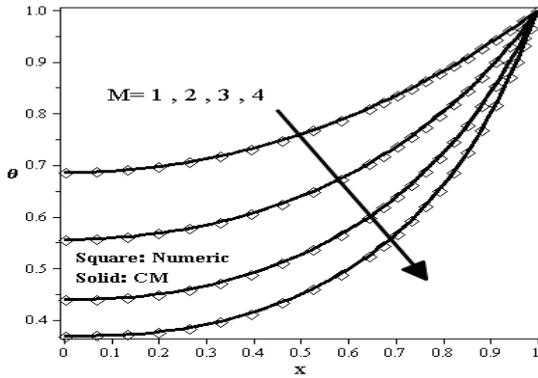


Figure 4. Effect of thermo-geometric fin parameter (M) on θ when $m = 1/4$, $\beta = 0.1$

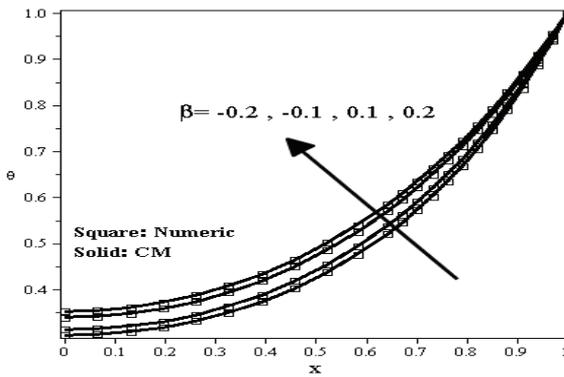


Figure 5. Effect of thermo-geometric fin parameter (β) on θ when $m = 1/4$, $M = 2$

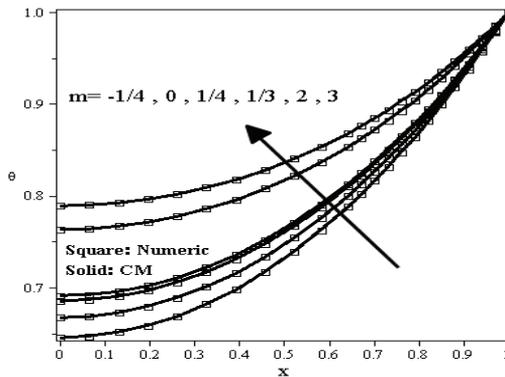


Figure 6. Effect of thermo-geometric fin parameter (β) on θ when $M = 1$, $\beta = 0.1$

4.0 CONCLUSION

In this paper, nonlinear heat transfer equations for fin with temperature-dependent thermal conductivity and heat transfer coefficient are presented and the CM has been successfully applied to find the most exact analytical solution. Furthermore, the obtained solutions by collocation method are compared with VIM, HPM and numerical solutions. The results demonstrate that the CM is powerful mathematical tools and has excellent agreement with numerical outcomes. Also accuracy of the solution can be increased by increasing the statements of the trial functions. The CM is very effective, simpler and offers superior accuracy compared with the variation iteration method and Homotopy Perturbation Method. It does not need any perturbation, linearization or small parameter versus Homotopy Perturbation Method and Variation iteration method.

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REFERENCES

- Kiwan, S. (2006). Thermal Analysis of Natural Convection Porous Fins. *Transport In Porous Media*, 67(1), 17-29.
- Gorla, R., & Bakier, A. (2011). Thermal analysis of natural convection and radiation in porous fins. *International Communications In Heat And Mass Transfer*, 38(5), 638-645.
- Domairry, G., & Fazeli, M. (2009). Homotopy analysis method to determine the fin efficiency of convective straight fins with temperature-dependent thermal conductivity. *Communications In Nonlinear Science And Numerical Simulation*, 14(2), 489-499.
- Domiri, G., Ziabksh, G., & Domiri, G. (2011). Determination of temperature distribution for annular fins with temperature dependent thermal conductivity by HPM. *Thermal Science*, 15(suppl. 1), 111-115.

- Khani, F., Raji, M., & Nejad, H. (2009). Analytical solutions and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient. *Communications In Nonlinear Science And Numerical Simulation*, 14(8), 3327-3338.
- Ganji, D.D., Hashemi Kachapi, S.H. (2011). Analytical and Numerical Methods in Engineering and Applied Sciences, *Progress in Nonlinear Science*, 3, 1-579.
- Ganji, D.D. Hashemi Kachapi. S.H. (2011). Analysis of Nonlinear Equations in Fluids. *Progress in Nonlinear Science*; 3, 1-294.
- He, J. (1999). Homotopy perturbation technique. *Computer Methods In Applied Mechanics And Engineering*, 178(3-4), 257-262.
- He, J. (2005). Homotopy Perturbation Method for Bifurcation of Nonlinear Problems. *International Journal Of Nonlinear Sciences And Numerical Simulation*, 6(2).
- He, J. (2005). Application of homotopy perturbation method to nonlinear wave equations. *Chaos, Solitons & Fractals*, 26(3), 695-700.
- Esmailpour, M., & Ganji, D. (2007). Application of He's homotopy perturbation method to boundary layer flow and convection heat transfer over a flat plate. *Physics Letters A*, 372(1), 33-38.
- He, J. (2010). A note on the homotopy perturbation method. *Therm. Sci.*, 14, 565-8.
- Ganji, D., & Rajabi, A. (2006). Assessment of homotopy-perturbation and perturbation methods in heat radiation equations. *International Communications In Heat And Mass Transfer*, 33(3), 391-400.
- Domiri, G., Ziabkhsh, G., & Domiri, G. (2011). Determination of temperature distribution for annular fins with temperature dependent thermal conductivity by HPM. *Thermal Science*, 15(suppl. 1), 111-115.
- He, J. (2007). Variational iteration method-some recent results and new interpretations. *J. Comput. Appl. Math.*, 207, 3-17.
- He, J. & Wu, X.H. (2006). Construction of solitary solution and compaction-like solution by variational iteration method. *Chaos Soliton. Fract.* 29, 108-13.
- Rostamiyan, Y., Ganji, D., Petroudi, R., & Nejad, K. (2014). Analytical investigation of nonlinear model arising in heat transfer through the porous fin. *Thermal Science*, 18(2), 409-417.

- Ganji, D.D., Tari, H., & Jooybari M.B. (2007). Variational iteration method and homotopy perturbation method for nonlinear evolution equations. *Comput. Math. Appl.*, 54, 1018-27.
- He, J. (1999). Variational iteration method-a kind of nonlinear analytical technique: Some examples. *Int. J. Nonlinear Mech.*, 34, 699-708.
- Stern, R.H. & Rasmussen, H., (1996). Left ventricular ejection: model solution by collocation, an approximate analytical method. *Comput Biol Med.*, 26, 255–61.
- Hu, H.Y. & Li, Z.C. (2006). Collocation methods for Poisson's equation. *Comput Method Appl Mech Eng.*, 195, 4139–60.
- Herrera, I., Diazviera, M. & Yates R. (2004). Single collocation point methods for the advection–diffusion equation. *Adv Water Resour.* 27, 311–22.
- Arnau, J.M., Company, R. & Rosello, M.D. (2004). A collocation method to compute onedimensional flow models in intake and exhaust systems of internal combustion engines. *Math Comput Model.* 40, 995–1008.
- Hendi, F.A. & Albugami, A.M. (2010). Numerical solution for Fredholm–Volterra integral equation of the second kind by using collocation and Galerkin methods. *J King Saud Univ.*; 22, 37–40.
- Hatami, M. & Ganji, D.D. (2013). Heat transfer and flow analysis for SA–TiO₂ non-Newtonian nanofluid passing through the porous media between two coaxial cylinders. *J Mol Liquids.* 188, 155–161.

