Journal of Engineering and Technology

ISSN 2180-3811 eISSN 2289-814X https://jet.utem.edu.my/jet/index

CASSON FLUID FLOW WITH HEAT SINK AFFECTED BY SYMMETRIC WALL TEMPERATURE AND CONCENTRATION CONDITIONS

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Article history:
Received Date: 6 June 2024
Revised Date: 6 September 2024
Accepted Date: 12 September 2024

Keywords: Casson Fluid Flow, Chemical Reaction, Heat Sink, Symmetric

Abstract— The problem of Casson fluid flow with heat sink impacted by symmetric wall temperature and concentration conditions has been investigated in this research. The leading equations of the formulated problem are solved analytically using the theory of simultaneous ordinary differential equations. The influence of several dimensionless factors relevant to the momentum, energy, mass diffusion, frictional force, heat transfer and mass transfer rates are investigated and depicted using illustrative graphs. It has been discovered that increasing the Casson fluid

Wall Concentration, Symmetric Wall Temperature.

and heat sink values significantly reduces the fluid's velocity and temperature. The existence of symmetric wall temperature and concentration substantially influences flow development and reversal. Casson fluids is commonly used in many notable technological and industrial properties, such as synthetic lubricants, specific oil paints, biological fluids, diverse polymer solutions to mention few. There are some features that defy comprehension when analyzed solely through lens of the Newtonian flow problem. Consequently, utilization of the non-Newtonian fluid motion is more productive.

I. Introduction

Heat transfer is a field within thermal science that encompasses the study of energy production, conversion, usage, and exchange in heat form within physical and mechanical systems. In essence, it can be described as the phenomenon through which thermal energy is transferred from one location to another as a result of a disparity in temperature. The phenomenon wherein mass is transferred from a premises of high concentration to a region of low concentration can be referred to as mass transfer. Currently, there is a significant amount of research being conducted worldwide on the principles of heat-mass transport, specifically focusing on the incorporation of non-Newtonian fluids. This research is prompted by the numerous practical applications and benefits that these concepts offer in our daily lives. Heat and mass transport problems are fundamental components of various scientific and technical applications, encompassing medicinal, physical, chemical, and biological domains. The term "Casson fluid" is employed to denote a type of non-Newtonian fluid characterized by variable viscosity. The dominance of viscous force in this system can be attributed to the action of variable viscosity in the fluid. Fluids commonly employed in many applications include paints, diverse polymer solutions, blood, honey, and other similar things. Nandeppanavar [1] described the movement pattern of a Casson fluid through a moving plate. The Casson liquid solution incorporates an exponential temperature-dependent heat generation/absorption and cross diffusion impacts. The study suggests that the fluid relaxation effect, when enhanced, leads to an embellishment in the velocity profile. In view of this, Zaib et al. [2] looked at the flow of Casson fluid along a permeable plate, paying special attention to how heat moved through the fluid. The research conducted by Hayat et al. [3] presents an analysis of the flow of Casson fluid on a plate's surface. Hussanan et al. [4] have conducted comprehensive investigations on the impact of a non-Newtonian fluid property on fluid flow. Sheikh et al. [5] presented a study on the implication of the flow of a non-Newtonian fluid across a moving sheet. Ahmad et al. [6] introduced a novel approach to modelling a Casson fluid with fractional derivatives in a more recent study. Arthur et al. [7] performed the analysis of Casson fluid flow, employing similarity technique to transform the underlying equations from basic and partial differential forms into linear ODEs. Further, Sarkar and Endalew [8, 9], Hamid et al. [10], Das et al. [11], Amjad et al. [12], and Sarkar et al. [13] have published the flow behavior of the Casson fluid in various physical settings. Several recent and noteworthy research [14-18] on this topic provide valuable insights for readers. Casson fluids have garnered significant attention among research scientists due to their notable technological and industrial properties. The Casson fluid is considered to be one of the most prominent types of fluids within the category of non-Newtonian substances. The substance in question is characterized by shear-thinning behaviour, whereby its viscosity approaches infinity at a zero-shear rate, indicating that no flow occurs beyond the yield stress. Additionally, the viscosity leads to zero as the frictional factors approaches infinity. Typical instances of Casson fluids encompass synthetic lubricants, mud extraction, clay coatings, specific, and biomedical fluids. The Casson fluid simulation that are readily accessible are classified based on their distinct rheological properties, including Oldroyd-B, Eyring-Powell, Cumbersome, Oldroyd-A, Maxwell, Carreau, Jeffrey, and Burger. The study of heat transfer characteristics of Casson fluid, employing heat radiation and permeability factors was conducted by Pramanik [19].

The investigation of the consequence of heat sinks on magnetohydrodynamic (MHD) flows is a source of inspiration for academics across other disciplines due to its wide-ranging industrial implications. The key function of heat absorption is to reduce thermal conductivity, resulting in a subsequent depression in the temperature of the fluid. The analysis of heat transmission from a non-uniform source/sink on a micropolar fluid over a stretching/shrinking sheet was investigated by Sandeep and Sulochana [20]. Ali et al. [21] demonstrated the involvement of heat sink/source, hydromagnetic flow, and heat transfer in a pair of stress fluids across a rotating moving plate that is immersed in a porosity influence. It has been observed that when the heat loads of contemporary portable electronic devices increase, the available surface area for heat dissipation diminishes. It is vital to address the escalating heat load of the gadget in order to prevent the occurrence of an overheating situation, which would adversely influence both the stability and functioning of the operational equipment. Due to the rapid increase in thermal load inside air cooling systems, it is imperative to improve the thermal management system, specifically the heat sink, in order to achieve optimal performance within the given spatial constraints. In recent decades, there has been a noticeable surge in scholarly attention towards the advancement of heat sink methods aimed at efficient heat dissipation. Consequently, numerous methodologies for optimizing heat sink designs have been formulated and established. In their study, Kyoungwoo Park [22] employed the Kriging method in conjunction with a computational fluid dynamics (CFD) approach to enhance the performance of a heat sink. Matthew and Stadler [23] discussed the difficulties associated with implementing fixed temperature boundary conditions for high-temperature surfaces. Additionally, he highlighted the importance of cellular materials in the maximization of heat absorptions. In a separate work, Konda et al. [24] examined the behaviour of a Williamson fluid as it flows along a vertical plate. The researchers specifically focused on the convective heat transfer occurring at the boundary wall and in the availability of irregular heat absorption/generation. Their findings revealed that the fluid temperature experiences a reduction in response to a rise in the heat absorbing parameter. Tsai et al. [25] employed the Chebyshev finite difference method to address the computational challenge of fluid flow over a deformable wall subjected to a nonuniform heat absorption/generation. In their study, Yousif et al. [26] inspected the characteristics of radiative Carreau nanofluid flow across an exponentially stretchable embedding. The investigation incorporated the impacts of MHD, radiation, and an internal heat absorption/generation. The findings revealed that uplifting the thermal radiation parameter led to an augmentation in the temperature distribution. Kumar and Varma [27] employed the Runge-Kutta technique to address the problem of fluid movement across a porous wall, considering the implications of suction and internal heat

ISSN: 2180-3811 Vol. XX No. X

generation /absorption. The research carried out by Abel et al. [28] discussed the actions of heat generation/absorption and MHD on the flow behaviour of Sisko fluid through a porous stretched sheet. In another related paper, Khan et al. [29] emphasized the impact of a heat emission/absorption on the flow behaviour of Maxwell nanofluids across a deformable surface. Irfan et al. [30] deliberated on the flow characteristics of viscoelastic fluid across a porous stretched wall in the involvement of MHD and a heat absorption/generation. In another article, Khan et al. [31] echoed the consequences of heat sink/source in hydromagnetic flow of Sisko fluid through a non-stationary permeable plate. Additionally, the researchers explored the behaviour of a MHD Maxwell nanofluid under the influence of a moving cylinder entrenched with a heat generation/absorption. In their study, Haq et al. [32] explained the action of heat absorption/generation and a chemically reactive fluid on the flow patterns of a nanofluid passing through a wedge-shaped cross-section. The flow and sensitivity analysis were enhanced by the inclusion of a chemical reaction in a mixed radiated magneto Casson fluid, as reported by Zahra et al. [33].

Based on the preceding discourse, the primary objective of the current study focuses on investigating the flow of Casson fluid with chemical reaction in a vertical plate, wherein the existence of heat sink, symmetric wall temperature and mass diffusion factors are also taking into account. The Casson fluid and chemical reaction significantly influence the transmission of heat and mass transfer. Shear stress and strain are nonlinearly related in the Casson model and this model can be used to study the blood flow, synthetic lubricants, paints industry and manufacturing of medicine.

II. Mathematical Analysis

The present study examines the heat and mass transfer flow of an electrically-conductive and incompressible fluid in a steadystate condition. The fluid is confined between two vertical plate walls, which are positioned at a distance L from each other, as seen in Figure 1. A coordinate system is selected in which the x-axis is oriented parallel to the gravitational force, but in the reverse direction. The y-axis is perpendicular to the walls of the channel, and the origin of the coordinate system is defined such that the dimensions of the channel walls are represented by y = 0 and y = L, respectively.

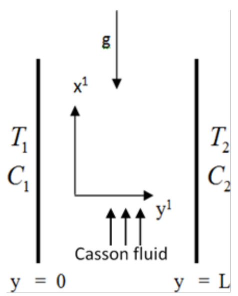


Figure 1: Sketch of the flow problem

Based on the research carried out by Ahmad et al. [34] and Pop et al. [35], it is posited that within the plate, energy is transferred to the nearby environment through a heat-releasing surface reaction characterized by a single first-order Arrhenius reaction. The leading equations in dimensional form can be expressed under the Boussinesq approximation.

$$v\left(1+\frac{1}{\xi}\right)\frac{d^2u'}{dy'^2}+g\beta\left(T'-T_0'\right)+g\beta^*\left(C'-C_0'\right)-\frac{1}{\rho}\frac{dP'}{dx'}=0 \quad (1)$$

$$\alpha\frac{d^2T'}{dy'^2}+Q_0K_0ae^{-E/RT'}-\frac{Q_1(T'-T_0')}{\rho C_p}=0 \quad (2)$$

$$D\frac{d^2C'}{dy'^2}=0 \quad (3)$$

Subject to the boundary conditions

$$u=0,\ T=T_1',\ C=C_1' \quad y'=0$$
$$u=0,\ T=T_2'\ C=C_2'\ y'=L \quad (4)$$

where, Q_0 is the exothermic, Q_1 is the temperature dependent volumetric rate of the heat sink, ρ is the density, β is the coefficient of heat expansion, T_0 and C_0 are the reference temperature and concentration respectively and assuming that $T_0=(T_1+T_2)/2$ and $C_0=(C_1'+C_2')/2$.

Introducing the following dimensionless quantities:

$$x=\frac{x'}{\mathrm{Re}\,L},\ y=\frac{y'}{L},\ u=\frac{u'}{u_0},$$

$$p=\frac{p'}{\rho u_0^{\,2}},\ \theta=\frac{T-T_0'}{RT_0^{\,2}/E},$$

$$\phi=\frac{C-C_0'}{RC_0'/E},\ \mathrm{Re}=\frac{u_0L}{v},$$

$$\lambda=\frac{Gr}{\mathrm{Re}},\ \gamma=\frac{dp'}{dx'},$$

$$\gamma_t=\frac{T_1'-T_0}{RT_0^{\,2}/E},\ \gamma_c=\frac{C_1'-C_0'}{RC_0^2/E},$$

$$K=\frac{EQK_0aL^2}{RT_0^2a}e^{-E/RT_0},$$

$$S=\frac{Q_1L^2}{K} \tag{5}$$

inserting (5) in to (1) to (4), the resultant ODEs as Equation (6), (7) and (8).

$$\left(1+\frac{1}{\zeta}\right)\frac{d^2u}{dy^2}+\lambda\left[\theta+N\phi\right]=\gamma \tag{6}$$

$$\frac{d^2\theta}{dy^2}+Ke^{\theta}-S\theta=0 \tag{7}$$

$$\frac{d^2\phi}{dy^2}=0 \tag{8}$$

Given boundary condition is as follow:

$$\begin{aligned}&u=0,\ \theta=\gamma_t,\ \phi=\gamma_c,\ \text{at}\ \ y=0\\&u=0,\ \theta=-\gamma_t,\ \phi=-\gamma_c,\ \text{at}\ \ y=1\end{aligned} \tag{9}$$

where, $\lambda, N, \gamma, K, \gamma_t, \gamma_c, \xi\ and\ S$ are mixed convection parameter, sustention parameter, pressure term, Frank-Kamenetskii number, wall temperature, wall concentration, Casson fluid parameter and Heat Sink parameter respectively.

III. Analytical Solution

For small value of $e^{\theta} \approx (1+\theta)$, Equations (6)-(8) restricted to (9) are determined by employing the theory of simultaneous differential equations. The expressions of momentum, energy and mass diffusion have been derived as Equations (11) - (13).

$$\begin{aligned}U&=B_1y+B_2+L_1y^2+\\&L_2y^3-L_3[A_1e^{y\sqrt{S-K}}+A_2e^{-y\sqrt{S-K}}]\end{aligned} \tag{11}$$

$$\theta=A_1e^{y\sqrt{S-K}}+A_2e^{-y\sqrt{S-K}}+A_3 \tag{12}$$

$$\phi=A_1y+A_2 \tag{13}$$

The expressions of Skin friction, heat and mass transfer rates at both plates are as Equations (14) – (19).

$$\begin{aligned}\left.\frac{du}{dy}\right|_{y=0}&=B_1-L_3A_1\sqrt{S-K}\\&+L_3A_2\sqrt{S-K}\end{aligned} \tag{14}$$

$$\left.\frac{du}{dy}\right|_{y=1} = B_1 + 2L_1 + 3L_2 - L_3 A_1 \sqrt{S-K} e^{\sqrt{S-K}} + L_3 A_2 \sqrt{S-K} e^{\sqrt{S-K}} \quad (15)$$

$$\left.\frac{d\theta}{dy}\right|_{y=0} = A_1 \sqrt{S-K} - A_2 \sqrt{S-K} \quad (16)$$

$$\left.\frac{d\theta}{dy}\right|_{y=1} = A_1 \sqrt{S-K} e^{\sqrt{S-K}} - A_2 \sqrt{S-K} e^{-\sqrt{S-K}} \quad (17)$$

$$\left.\frac{d\phi}{dy}\right|_{y=0} = -2\gamma_c \quad (18)$$

$$\left.\frac{d\phi}{dy}\right|_{y=1} = -2\gamma_c \quad (19)$$

where, $A_1, A_2, A_3, B_1, B_2, L_1, L_2,$ *and* L_3 are constants which are indicated in the appendix.

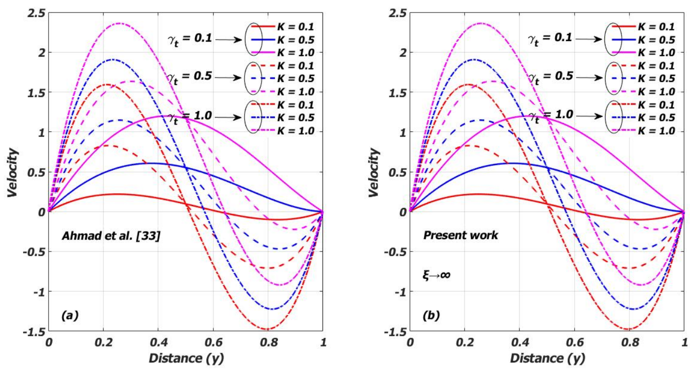
IV. Results and Discussion

Mixed convection Casson fluid flow with heat sink has been examined in a vertical channel with symmetric wall temperature and concentration conditions. The current study's results are graphed from Figure 2 to Figure 13. The parameters that regulate the flow systems are as follows.: the Casson parameter (ξ), Frank-Kamenetskii parameter (K), mixed convection parameter ($\lambda = \frac{Gr}{\mathrm{Re}}$), Sustentation parameter (N), heat sink parameter (S), wall temperature (γ_t) and wall concentration (γ_c). The following range of values are used: $0.01 \le \xi \le 0.03$, $2 \le S \le 6$, $0.1 \le K \le 1.5$, $0.1 \le \gamma_t \le 5$, $0.1 \le \gamma_c \le 5$, $0.01 \le N \le 0.1$ and $\lambda = 100$.

Figure 2 illustrates the contrast between the velocity profile analysis conducted by Ahmad et al. [35] and the current study, and portray the relationship between the Frank-Kamenetskii parameter (K) and various values of γ_t . However, it is observed in Figure 2(b) that as ξ approaches infinity, there is relatively very insignificant influence of ξ on the flow that significantly boost fluid velocity profile at the cold plate but a reverse phenomenon happens at the hot plate.

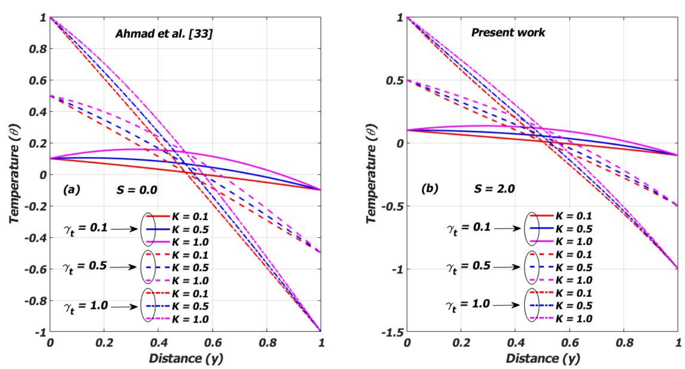
Figure 3 represents the comparison between the work of Ahmad et al. [34] and the current work on temperature distribution. Figure 3(a) represents the role of Frank –

Kamenetskii parameter (K) with respect to different values of γ_t in the absent of heat sink (S), while Figure 3(b) represent the role of Frank – Kamenetskii parameter (K) with respect to different values of γ_t under the action of heat absorption (S). It is important to note that the inclusion of a heat sink (S) in Figure 3(b) leads to the suppression of the temperature distribution. The underlying cause of this behavior is the absorption of heat from the surrounding channel, resulting in a commensurate decrease in the fluid temperature.



(a) (b)

Figure 2: Comparison between the work of Ahmad et al. [34] and the present work on Velocity profile

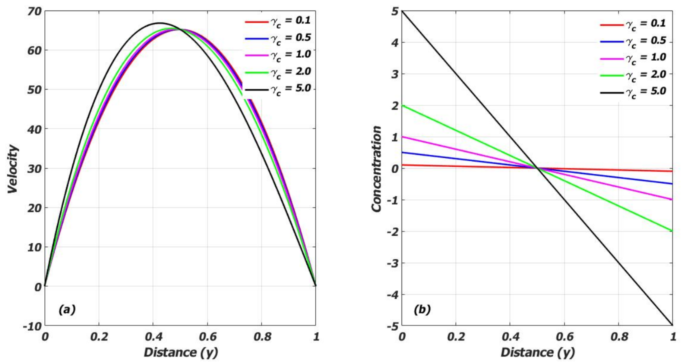


(a) (b)

Figure 3: Comparison between the work of Ahmad et al. [34] and the current work on Temperature profile

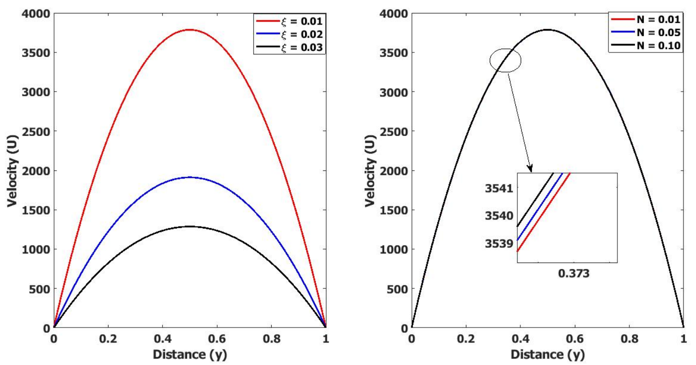
Figure 4 illustrates the impact of γ_c on both velocity and concentration of the fluid. It reveals that increasing γ_c lead to an enhancement in the concentration of the fluid at the lower plate (y = 0) whereas an opposite phenomenon is noticed at the upper plate (y = 1). As expected at the bottom plate, the channel surface is heated while at the upper plate the channel wall is cooled. It is evident from Figure 4 that, there exist a point of intersection at the middle of the channel.

Figure 5 illustrates the fluid motion distribution for different values of ξ and N. It is concluded from Figure 5 that as ξ rises, the velocity profile decreases while as N is improved, the velocity gradient remains fixed.



(a) (b)

Figure 4: Concentration profile for $\xi = 0.01$ and $S = 2.0$ for different values of (γ_c)

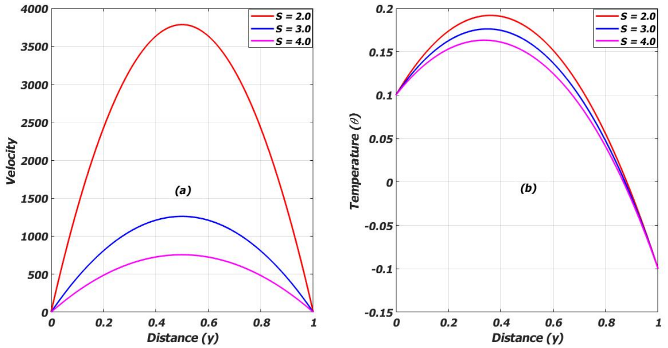


(a) (b)

Figure 5: Velocity profile for $\lambda = 100$ with different values of $\xi and N$

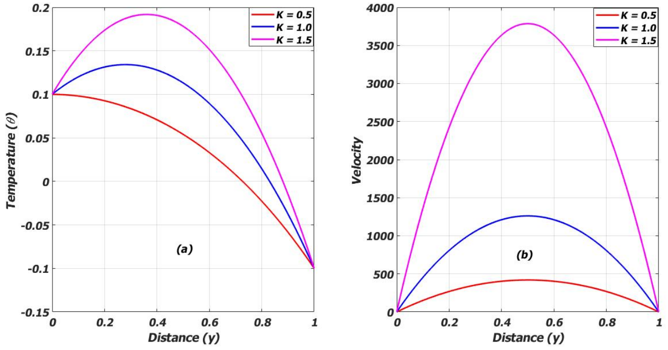
Figure 6 illustrates the function of the heat sink (S) on the velocity and temperature profiles. The findings indicate that there is a notable deceleration in fluid velocity and temperature with a rise in the heat sink (S) factor. Higher values of S indicate that heat is absorbed from the fluid, resulting in a substantial reduction in both temperature and velocity distributions.

Figure 7 showcases the impact of the Frank-Kamenetskii parameter (K) on the temperature and velocity patterns. The figure illustrates that higher values of K have a substantial positive impact on both the temperature and velocity of the fluid. The generation of heat is significantly increased for higher values of K due to the occurrence of an exothermic reaction within the channel.



(a) (b)

Figure 6: Velocity and Temperature profiles for $\lambda = 100$ and different values of S

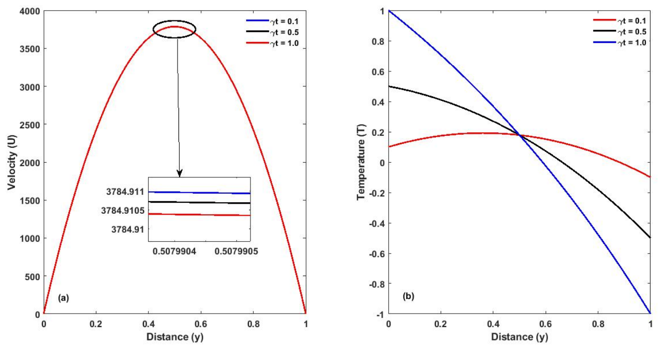


(a) (b)

Figure 7: Temperature and Velocity profiles for $\lambda = 100$ and different values of K

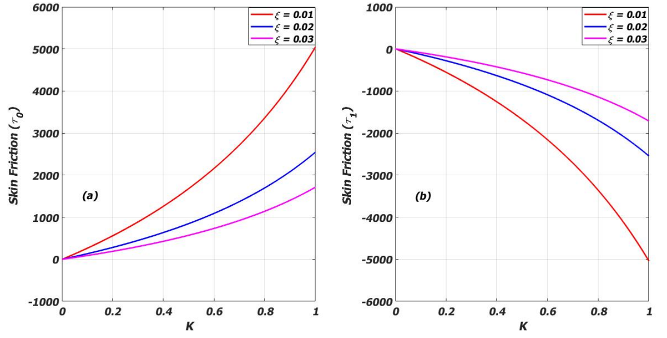
The effect of wall temperature (γ_t) on both velocity and temperature distributions is shown on Figure 8. Higher values of γ_t increases velocity of the fluid. Velocity of fluid is seen to be higher at the center of the channel. At lower plate, temperature is enhanced while the opposite trend is noticed at upper plate. Symmetric heating of the plate significantly contributed to the occurrence of this behavior. Also, a point of intersection is noticed in the middle region of the channel.

Figure 9 illustrates the relationship between the wall shear stress and the variable K for different values of ξ In Figure 9(a), it can be shown that a weakening in skin friction exist as the values of increase. Conversely, it is evident that skin friction is higher when the value of K is big. Figure 9(b) exhibits contrasting behavior when the value of y is equal to 1.



(a) (b)

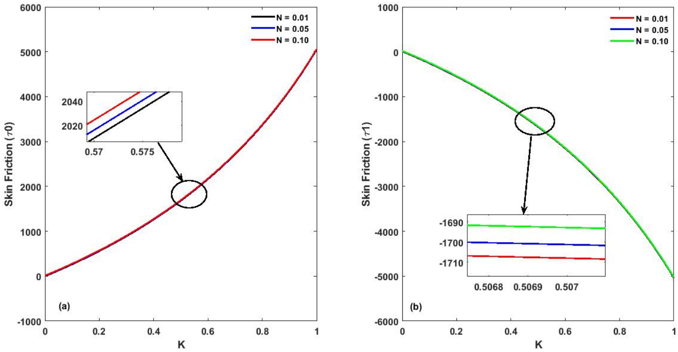
Figure 8: Temperature profiles for $\lambda = 100$ and different values for γ_t



(a) (b)

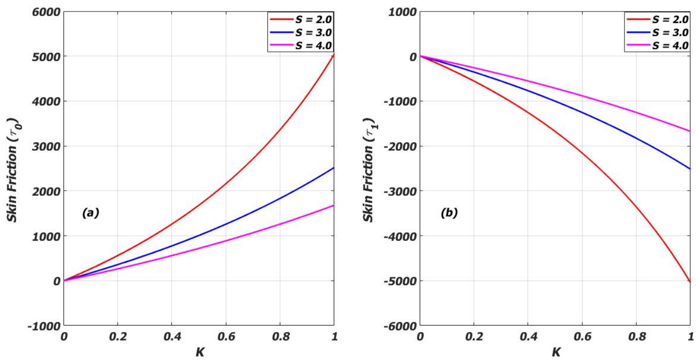
Figure 9: Skin friction at y = 0 and y = 1 with different values of ξ

Figure 10 presents the relationship between skin friction and the variable K for different values of N. The skin friction at both the lower (y = 0) and upper (y = 1) plates is observed to increase as the values of N increase. Figure 11 illustrates the influence of a heat absorption (S) on skin friction. The skin friction diminishes as the values of S increase at the cold plate (y = 0). A contrasting phenomenon is observed at the upper plate located at y = 1.



(a) (b)

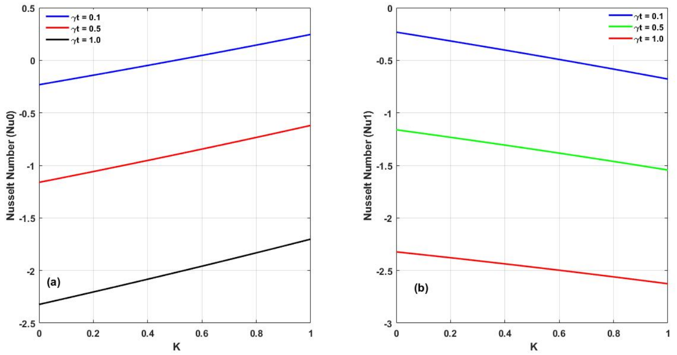
Figure 10: Skin friction at y = 0 and y = 1 with different values of N



(a) (b)

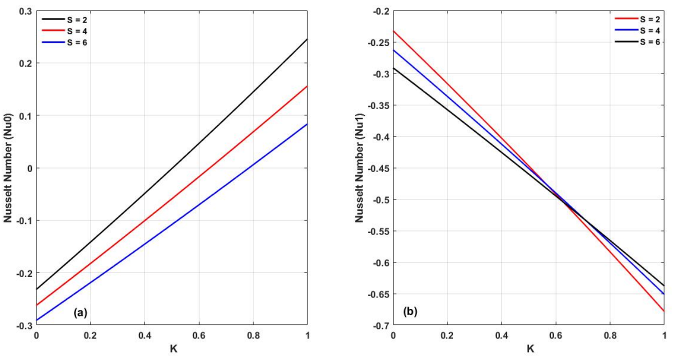
Figure 11: Skin friction at y = 0 and y = 1 with different values of S

Figures 12 depicts the impact of temperature differential on the heat transfer coefficient at two distinct plates, namely y = 0 and y = 1, respectively. It is noticed that increasing γ_t reduces the heat transfer amount but mounting values of K it seen to improve the heat transfer amount. Figure 13 is sketched to see the impact of heat generation on the heat transfer rate at both plates y = 0 and y = 1 respectively. Higher values of S significantly reduce the heat transfer coefficient.



(a) (b)

Figure 12: Nusselt Number at y = 0 and y = 1 with different values of γ_t



(a) (b)

Figure 13: Nusselt Number at y = 0 and y = 1 with different values of S

V. Conclusion

This paper examines a class of fluids known as non-Newtonian fluid flow where stress and viscosity variations are independently related and have several interesting applications in practical existence such as in the mining industry, metallurgy, nanotechnology, material science and food processing. The influence of symmetric wall temperature and concentration conditions with heat sink are also considered. The velocity and temperature distributions are derived analytically by the application of the theory of simultaneous ordinary differential equations. This study investigates the impact of many parameters, specifically the Frank-Kamenetskii parameter (K), sustentation parameter (N), Casson fluid flow, and heat sink (S). The study's findings can be summarized as follows:

(i) It has been concluded that raising the Casson fluid parameter (ξ) results in a deceleration in the velocity distribution of the flow.

(ii) It has been revealed that an increase in the heat sink parameter (S) results in a notable reduction in the temperature and velocity distributions of the flow.

(iii) The velocity and temperature of the fluid are strongly influenced by the action of symmetric wall temperature and concentration. Flow reversal is observed as the symmetric wall temperature and concentration increase.

VI. Appendix

$$A_1 = \gamma_t - A_2 - A_3$$

$$A_2 = \frac{A_3[e^{\sqrt{S-K}} - 1] - \gamma_t[1 + e^{\sqrt{S-K}}]}{[e^{-\sqrt{S-K}} - e^{\sqrt{S-K}}]}$$

$$A_3 = -\frac{K}{[K-S]}$$

$$B_1 = -L_3 A_1 - L_3 A_2 - L_1 - L_2$$
$$+ L_3 A_1 e^{\sqrt{S-K}} + L_3 A_2 e^{-\sqrt{S-K}}$$

$$B_2 = L_3 A_1 + L_3 A_3$$

$$L_1 = \frac{\gamma}{2\alpha} - \lambda \frac{A_3}{2\alpha} - \lambda \frac{N\gamma_c}{2\alpha}$$

$$L_2 = \lambda \frac{N\gamma_c}{3\alpha}, \quad L_3 = \frac{\lambda}{\alpha(S-K)}$$

VII. Acknowledgement

The first author conceptualized the problem and provided the methodology. The second author wrote the main draft of the manuscript. The third author contributed to the analysis, while the fourth author performed the validation, and the fifth author contributed to the editing of this work. All authors read and approved the final manuscript.

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