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INFLUENCE OF MAGNETIC FIELD-DEPENDENT VISCOSITY ON MHD MIXED MAGNETISED FLUID OVER A MOVING SHEET

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Abstract— Nanotechnology encourages the biocompatibility of hybrid ferrofluids (HFFs). The present research investigates the flow of a HFF composed of ferrite (Fe_3O_4) and cobalt ferrite $(CoFe_2O_4)$ dissolved in a mixture of ethylene glycol with water (EG/H_2O) solution past a moving sheet. The mixed convection is considered. The presence of the magnetohydrodynamics (MHD) effect controls the flow of the HFF, leading to the development of the magnetic field-dependent viscosity (MFDV). The problem is governed by partial differential equations (PDEs), which are then reduced to ordinary

Mixed convection, bvp4c	differential equations (ODEs) using similarity variables. The bvp4c package in MATLAB is employed to solve the resulting ODEs. The findings show that enhancing the MFDV results in higher velocity and temperature patterns, while reducing the shear stress and heat transfer rate by 21.17% and 6.88%, respectively. The temperature of the HFF decreases due to variations in the mixed convection parameter. The outcomes of the present study can serve as a useful guideline for developing ferrite-based nanoparticles for biomedical applications.

I. Introduction

Nanotechnology constitutes a driving force in the advancement of FFs to hybrid ferrofluids (HFFs). HFFs refer to a mixture of ferrite-based nanoparticles and other nanoparticles, which can be either magnetic or non-magnetic, that are dissolved in a conventional fluid [1]. Zainodin et al. [2] have conducted a study on the mixed convection of HFF, magnetite ferrite (Fe_3O_4) and cobalt ferrite ($CoFe_2O_4$) with water (H_2O) over a nonlinearly moving surface and the magnetohydrodynamics (MHD) effect. They discovered that the addition of $CoFe_2O_4$ to $Fe_3O_4/CoFe_2O_4$ has enhanced the heat and mass transfer when compared to FF, Fe_3O_4 - H_2O and H_2O . Idris et al. [1] investigated the behaviour of $Fe_3O_4/CoFe_2O_4$ with viscous dissipation effect along with moving sheet. They obtained findings comparable to those of Zainodin et al. [2]. The HFF of $Fe_3O_4/CoFe_2O_4$ dissolved in a 50% EG: 50% H_2O mixture has been shown to have a larger heat transfer. Moreover, Ramesh et al. [3] acknowledged an augmentation in the thermal efficiency of $Fe_3O_4/CoFe_2O_4$ by incorporating the ferrite-based nanoparticles into a mixture of H_2O (50%) and EG (50%). Intriguingly, the stability analysis showed the unexpected stability of the EG/H_2O mixture.

In addition, Esmaeilnezhad et al. [4] argued that the interaction between magnetic fields and ferrite nanoparticles leads to significant enhancements in properties such as viscosity and yield stress. This phenomenon is known as the magnetic field-dependent viscosity (MFDV) effect [5]. Mehryan et al. [6] investigate the effect of MFDV in the mixtures of multi-walled carbon nanotubes (MWCNT) and Fe_3O_4. They observed a notable influence of the MFDV effect on the HFF Nusselt number over a porous medium. Izadi et al. [7] found that the MFDV effect enhances the velocity and temperature fields of micropolar FF. Furthermore, An increase in the resistance force in fluid flow due to the MFDV parameter decays the deformation of shear stress along with a rotating disc [8].

Drawing from the references discussed above, Izadi et al. [7] reported on the MFDV and MHD effects on the FF model. However, the mixed convection of HFF was not considered. Additionally, it is evident that there is a lack of research focusing on the MFDV effect specifically in HFF flow. To address this gap, the current study investigates the flow of the HFF with the MFDV under the influence of the MHD effect [6, 7]. This study considers the mixed convection of Fe_3O_4/ $CoFe_2O_4$ dissolved in a mixture of EG with H_2O [2, 3]. The partial differential equations (PDEs) that represent the MHD HFF with the MFDV are initially formulated. Appropriate similarity variables are used to transform the PDEs into a system of ordinary differential equations (ODEs). The BVP4C method is applied to obtain numerical solutions. The parameters being examined are assessed following the visualization of the temperature and velocity distributions, as well as important engineering parameters.

II. Mathematical Formulation

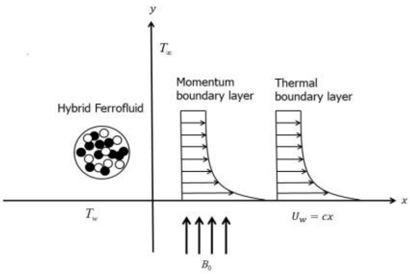

Figure 1: Physical Geometry [1, 7]

The HFF is modelled as incompressible, Newtonian, steady, and laminar viscous fluid over a moving sheet. The HFF flow velocity components are represented by (u, v) in respect to the (x, y) axis, respectively, as shown in Figure 1 [1, 7]. The sheet is moved in the x-axis direction with a velocity of $U_w = cx$, where $c > 0$ represents the rate of moving sheet required to initiate the movement of the fluid. y-axis is at a right angle to the sheet [1, 7]. The magnetic field B_0 is oriented at a right angle to the x-axis (see Figure 1). The sheet remains at a consistent temperature (T_w), while T_∞ represents the ambient temperature of the HFF. The equations that govern the two-dimensional HFF flow are expressed as Equation (1) to (5) [1, 2, 7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned} \rho_{hff}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) &= \eta_{hff}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma_{hff}B_0^2 u \\ &+ g(\rho\beta)_{hff}(T - T_\infty), \end{aligned} \tag{2}$$

$$(\rho C_p)_{hff}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{hff}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \eta_{hff}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$y = 0:\ u = U_w, v = 0, T = T_w, \tag{4}$$

$$y \to \infty:\ u \to U_\infty, T \to T_\infty. \tag{5}$$

The terms η_{hff} represents the MFDV effect in both Equations (2) and (3). It is defined as $\eta_{hff} = \mu_{hff}(1 + \boldsymbol{\delta} \cdot \boldsymbol{B})$ where $\boldsymbol{B}$ consists of the magnetic field components B_x and B_y in the $x-$ and $y-$ directions, respectively. According to

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Figure 1, the magnetic induction vector is given by $B_x = 0$ and $B_y = B_0$, indicating that the magnetic field exists only in the $y-$direction with no component in the $x-$ direction. Additionally, the vector δ refers to the MFDV vector components δ_x and δ_y in the $x-$ and $y-$ directions, respectively. Since the MFDV effect is assumed to be uniform in the two-dimensional flow considered in this study, it follows that $\delta = \delta_x = \delta_y$. Thus, the expression simplifies to $\eta_{hff=}\mu_{hff}(1+\delta B_0)$ [7]. Equations (4) and (5) represent the boundary conditions. The term with the subscript of hff refers to the HFF and can be established in [1]. The thermophysical characteristics of the conventional fluid with ferrite-based nanoparticles are detailed in [9-10]. The similarity variables are introduced as Equation (6) [7].

$$\psi(\eta,\xi) = (c\nu_f)^{\frac{1}{2}} x f(\eta),$$
$$\eta = \left(\frac{c}{\nu_f}\right)^{\frac{1}{2}} y, \qquad (6)$$
$$\theta(\eta) = \frac{T - T_\infty}{T_c - T_\infty}.$$

The symbol ν_f represents the kinematic viscosity of the fluid, which is calculated as the ratio of the dynamic viscosity (μ) to the density (ρ). The function $\psi(\eta,\xi)$ depicts the stream function of the flow. The components (u,v) can be described as $u = cxf'$ and $v = (c\nu_f)^{\frac{1}{2}} f$. The velocity equations are consistent with Equation (1). $\theta(\eta)$ is the dimensionless variable of the temperature [7]. Using Equation (6), Equations (1) to (5) are simplified to Equations (7) to (10).

$$\begin{aligned}(1+\delta_{mv})f''' &+ \left(\frac{\mu_{cf}}{\mu_{hff}}\right)\left(\frac{\rho_{hff}}{\rho_{bf}}\right)[ff'' - f'^2] \\ &- \left(\frac{\sigma_{hff}}{\sigma_{cf}}\right)\left(\frac{\mu_{cf}}{\mu_{hff}}\right)\mathrm{M}f' \\ &+ \left(\frac{\mu_{cf}}{\mu_{hff}}\right)\left(\frac{(\rho\beta)_{hff}}{(\rho\beta)_{cf}}\right)\chi\theta(\eta) = 0,\end{aligned} \qquad (7)$$

$$\left(\frac{k_{hff}}{k_{cf}}\right)\frac{1}{Pr}\theta'' + \left(\frac{(\rho C_p)_{hff}}{(\rho C_p)_{cf}}\right)\theta' f + \left(\frac{\mu_{cf}}{\mu_{hff}}\right)(1+\delta_{mv})\lambda f''^2 = 0, \tag{8}$$

$$\text{at } \eta = 0:\ f(\eta) = 0,\ f'(\eta) = 1, \theta(\eta) = 1, \tag{9}$$

$$\text{at } \eta \to \infty:\ f'(\eta) \to 0, \theta(\eta) \to 0. \tag{10}$$

The embedded essential dimensionless parameters are MFDV effect, MHD effect, Prandtl number, Eckert number, mixed convection parameter defined as $\delta_{mv} = \delta B_{0,}$ $M = \frac{\sigma_{cf}}{\rho_{cf} c} B_0^2$, $Pr = \frac{(\mu C_\rho)_{cf}}{k_{cf}}$, $\lambda = \frac{U_w^2}{(C_\rho)_{cf}(T_w - T_\infty)}$, $\chi = \frac{g\beta_{cf}(T_w - T_\infty)x^3}{\nu^2}$, respectively. The important physical factors, namely the skin friction (C_{fx}) and Nusselt number (Nu_x), in relation to the Reynold number $\left(Re = \frac{xU_w}{\nu_f}\right)$ are [1]:

$Re_x^{-\frac{1}{2}} Nu_x = -\frac{k_{hff}}{k_{cf}}[\theta'(0)]$ and

$C_f Re_x^{\frac{1}{2}} = -\frac{\mu_{hff}}{\mu_{cf}} f''(0)$.

III. Results and Discussion

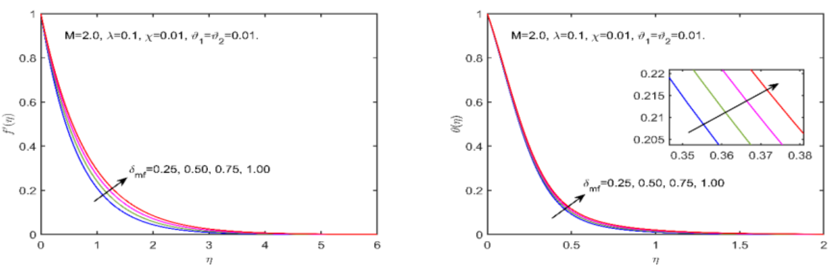
The MATLAB bvp4c solver is implemented to numerically solve ODEs (Equations (7) and (8))subject to the corresponding constraints Equations (9) and (10). Table 1 demonstrates the code validation findings for the Prandtl number (Pr) . The computed Nusselt number $(\theta'(0))$ in the current study is identical to the findings reported by Izadi et al. [7], in which the Runge-Kutta method was used to determine the results. The comparison enhances the dependability of the computations and evaluations for the remaining assignments in this investigation.

Table 1: Validation of Pr on $-\theta'(0)$ under the conditions $M = \delta_{mf} = \vartheta_1 = \vartheta_2 = \lambda = \chi = 0$

Pr	Izadi *et al.* [7]	Current study
1.00	0.582010	0.583866
5.00	1.568051	1.567866
10.0	2.308001	2.307846
100	7.765490	7.765537

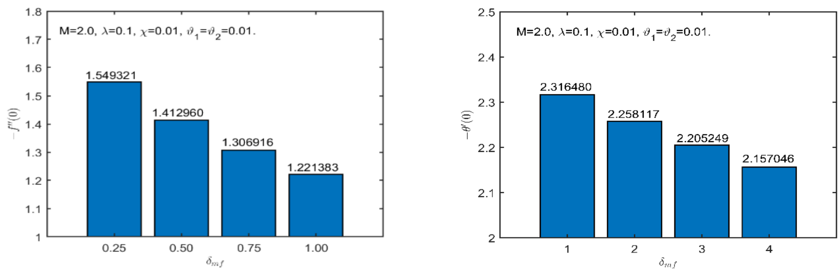
The effect of MFDV (δ_{mf}) over the velocity field $(f'(\eta))$ is portrayed in Figure 2(a). It can be seen that an enhancement of the parameter δ_{mf} results in boosting the profile of $f'(\eta)$. According to the MFDV equation, $\mu_{hff}(1 + \delta B_0)$ where $\delta B_0 = \delta_{mf}$, the growth of δ_{mf} leads to a declination in the fluid viscosity (μ_{hff}). This decreases in μ_{hff} causes an increase in $f'(\eta)$, enabling the HFF to move with greater freedom and boosting the thickness of the momentum boundary layer. The variation of the temperature profile $(\theta(\eta))$ with the parameter δ_{mf} is illustrated in Figure 2(b). It is observed that $\theta(\eta)$ enhances, as the parameter δ_{mf}. When the parameter δ_{mf} is intensified, the interaction between the HFF and magnetic fields strengthens due to the magnetic properties of the fluid.

This enhanced interaction generates additional heat within the hybrid ferrofluid. As a result, the HFF temperature and thermal boundary layer thickness rise due to enhanced heat retention in the fluid. It is depicted that the MFDV effect within the HFF flow has a prominent effect in $f'(\eta)$ compared to $\theta(\eta)$. Both profiles demonstrate similar patterns to the previous research reported by Izadi et al. [7].



(a) (b)

Figure 2: Variation of (a) $f'(\eta)$ and (b) $\theta(\eta)$ as a function of the parameter δ_{mf}



(a) (b)

Figure 3: Variation of (a) $f''(0)$ and (b) $\theta'(0)$ as a function of the parameter δ_{mf}

Figure 3(a) displays the pattern of skin friction $\left(f''(0)\right)$ with increasing the parameter δ_{mf}. The behaviour of $f''(0)$ is reduced, as the parameter δ_{mf} upsurges. This reduction occurs due to an increase in $f'(\eta)$, indicating that the fluid flows more freely. Consequently, the interaction between the moving wall and the HFF weakens, as the frictional forces at the boundary diminish. This leads to a significant decrease in the shear stress at the surface. The effect of parameter δ_{mf} on Nusselt number $(\theta'(0))$ is presented in Figure 3(b). It is demonstrated that an increase in the parameter δ_{mf} results in reducing $\theta'(0)$. The enhancement of the MFDV effect is represented by the parameter δ_{mf} enhances the conductive heat transfer than the convective heat transfer rate. When the parameter δ_{mf} increases, more heat is transferred from the sheet to the fluid. As a result, the overall heat transfer rate is reduced. The proportion of $f''(0)$ and $\theta'(0)$ are declined by 21.17% and 6.88%, respectively.

Figure 4(a) reveals the behaviour of $f'(\eta)$ with respect to the various values of the parameter χ. It is detected that an increase in the parameter χ results in a surge in $f'(\eta)$. This occurs because the parameter χ has a direct relationship with the buoyancy force, while it has an opposite relationship with the viscous force. The buoyancy force dominates over the viscous force when the parameter χ

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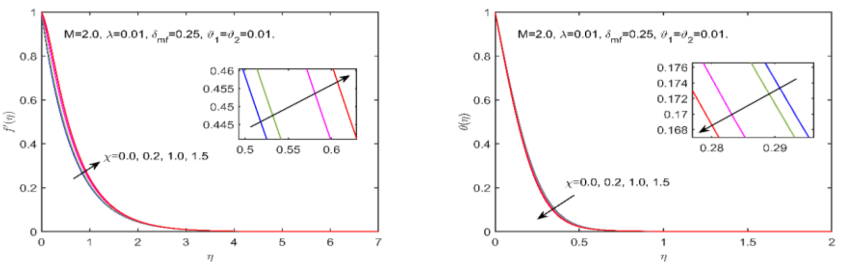
intensifies. Consequently, the hybrid ferrofluid flows more freely, and the momentum boundary layer thickness increases. The behaviour of $\theta(\eta)$ due to the parameter χ is portrayed in Figure 4(b). An increase in the parameter χ causes $\theta(\eta)$ to decline. This corresponds to the increased $f'(\eta)$, which enhances convection within the fluid, while conduction between the fluid and the wall is reduced. Consequently, the fluid temperature and the thermal boundary layer thickness were reduced.



(a) (b)

Figure 4: Variation of (a) $f'(\eta)$ and (b) $\theta(\eta)$ as a function of the parameter χ

IV. Conclusion and Recommendations

The study demonstrates a direct relationship between the velocity and temperature distributions of the HFF and augmentation in the MFDV over the moving sheet. The MFDV parameter exerts a greater influence on the velocity field compared to the temperature field. Additionally, the introduction of the mixed convection parameter lessens the temperature field and elevates the velocity profile as a result of the buoyancy force exerted on the HFF flow. As the MFDV grows, there is a considerable drop in both the heat transfer rate and skin friction. The inclusion of MFDV, and mixed convection characteristics is essential in managing the HFF system, offering potential benefits in optimizing the circulation of HFF during hyperthermia cancer therapy.

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